The Interplay among Instructional Strategies, Learners' Geometry Attitudes, and Van Hiele Levels of Geometric Reasoning: Evidence from Secondary Schools in Rwenzori Sub-Region, Uganda

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DECLARATION AND APPROVAL

CANDIDATE'S DECLARATION

I, Issa Ndungo, declare that the work presented in this Ph.D. thesis, titled *"The Interplay among Instructional Strategies, Learners' Geometry Attitudes, and Van Hiele Levels of Geometric Reasoning: Evidence from secondary Schools in Uganda's Rwenzori Sub-Region,"* is the result of my original research endeavors. Any contributions from external sources, such as publications, have been appropriately cited and referenced. The work has not been previously submitted for any academic award, and I take full responsibility for any errors or omissions in this document. By signing below, I affirm the accuracy and authenticity of this work.

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DECLARATION ON PUBLISHED WORK

This thesis incorporates materials that have previously appeared in peer-reviewed journal articles authored by the candidate as part of the research dissemination process. These publications were developed within the scope of this doctoral research, and while sections have been adapted or expanded to maintain coherence and thematic consistency, the findings and interpretations remain integral to the overall objectives of the thesis.

The relevant publications are:

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DEDICATION

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ABSTRACT

This study addresses persistently low achievement in geometry among Ugandan secondary school learners, alongside limited technology integration and underuse of learner-centered instructional strategies. Anchored in the pragmatic paradigm, Van Hiele theory of geometric reasoning, and constructivist learning theory, it examined the interplay between instructional strategies, learner attitudes, and progression through the Van Hiele levels in transformation geometry in the Rwenzori sub-region.

A quasi-experimental pre-test–post-test design was implemented over six weeks with 483 Senior Three learners from six secondary schools (three rural, three urban), comparing Conventional Van Hiele Phased Instruction (CVHPI, n = 245) and Technology-Enhanced Van Hiele Phased Instruction using GeoGebra (TVHPI, n = 238). This is among the earliest mixed-methods studies in Uganda to compare these approaches, providing evidence-based insights for curriculum reforms.

The first objective investigated changes in learners' attitudes toward geometry using a validated 22-item scale (α = 0.69–0.78). Wilcoxon Signed-Rank and Mann-Whitney U tests, alongside a Difference-in-Differences OLS model, revealed significant attitude gains in both groups ($p < 0.001$). However, TVHPI yielded consistently greater improvements, especially in affective domains (interest and confidence) and behavioral aspects (engagement and metacognition) ($\beta_3 = 0.24$, $p < 0.001$).

The second objective assessed geometric reasoning through the Van Hiele Geometry Achievement Test (α = 0.781). Both groups progressed significantly through Van Hiele levels ($p < 0.001$), but Generalized Ordered Logit results indicated that TVHPI learners were nearly twice as likely as CVHPI learners to achieve higher reasoning levels (OR = 1.94, $p < 0.001$). A significant interaction effect (β = 0.503, p = 0.034) reinforced the superior cognitive gains associated with technology integration.

The third objective explored whether attitudes predicted or mediated Van Hiele progression using a Difference-in-Differences–Generalized Ordered Logit model. Neither pre-instruction (β = 0.0155, p = 0.911) nor post-instruction attitudes (β = 0.0889, p = 0.631) significantly predicted progression, and attitude change did not mediate the instructional effect (β = –0.0997, p = 0.565). Instructional strategy remained the dominant predictor, though a marginal interaction (β = –0.6210, p = 0.075) suggested potential for stronger attitudinal influence in CVHPI contexts.

The fourth objective qualitatively examined learner experiences through semi-structured interviews with 48 purposively selected learners. Thematic analysis revealed two overarching themes: *Challenges and Support Needs,* and *Instructional Effectiveness and Learner Satisfaction*. TVHPI learners commended GeoGebra for its dynamic visualization for boosting engagement, conceptual understanding, and problem-solving confidence. CVHPI learners valued accessible local visual aids but noted slower mastery and heavier reliance on repeated practice. Common challenges included time constraints, multi-step problem difficulty, and limited resources.

Collectively, the findings affirm the Van Hiele theory's value in structuring geometry learning and underscore the constructivist advantage of technology-enhanced visualization for attitudinal and cognitive gains. The study culminated in the Ndungo Framework, the Geometry Pedagogical Improvement Cycle (GeoPIC), a blended, teacher-guided model combining the strengths of CVHPI and TVHPI.

It concludes that integrating tools like GeoGebra into Van Hiele Phased Instruction significantly enhances reasoning progression and attitudes. Recommendations include testing GeoPIC effectiveness, embedding technology-enhanced geometry pathways in the national curriculum, revising teacher training to emphasize competency-based, technology-supported pedagogy, and conducting further research on long-term effects and scalability across mathematical domains.

ACRONYMS AND ABBREVIATIONS

Acronym/Abbreviation	Full Meaning
ACH	Achievement
ATT	Attitude
BAC Model	Behavioral, Affective, and Cognitive Model
CBC	Competency-Based Curriculum
CBGL	Case-Based Game Learning
CVHPI	Conventional Van Hiele Phased Instruction
CVS	Comma-separated Values
DiD	Difference-in-Differences
EFA	Exploratory Factor Analysis
GeoGebra	Geometry & Algebra Software
GeoPIC	Geometry Pedagogical Improvement Cycle
GOL	Generalized Ordered Logit
H_a	Alternative hypothesis
H_o	Null hypothesis
ICT	Information and Communication Technology
KDE Plots	Kernel Density Estimate plot
KMO	Kaiser-Meyer-Olkin
MMU	Mountains of the Moon University
MUST	Mbarara University of Science and Technology
NAPE	National Assessment of Programs in Education
NCDC	National Curriculum Development Centre
NCST	National Council for Science and Technology
OLS	Ordinary Least Squares
ORs	Odds Ratios
REACT	Relating, Experiencing, Applying, and Transferring

Acronym/Abbreviation	Full Meaning
REC	Research Ethics Committee
RME	Realistic Mathematics Education
S.3	Senior three
SD	Standard Deviation
SEM	Structural Equation Modeling
TGAS	Transformation Geometry Attitude Scale
TVHPI	Technology-Enhanced Van Hiele Phased Instruction
UCE	Uganda Certificate of Education
UNEB	Uganda National Examination Board
VGAT	Van Hiele Geometry Achievement Test
VHPI	Van Hiele Phased Instruction
ZPD	Zonal of Proximal Development
ΔTGAS	Change in Attitude scores

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CHAPTER ONE

INTRODUCTION

In this chapter, the background of the study is presented in four subsections, beginning with a historical overview, followed by the conceptual, the theoretical perspective, and the contextual perspective. The chapter then proceeds to the problem statement, defining the study's purpose, specific objectives, and research questions/hypotheses. Lastly, the key terms, the justification of the study, and its scope are highlighted.

1.1 Background

This section presents the background to the study through four perspectives that collectively frame its context and purpose. The historical perspective traces the evolution of geometry instruction and the enduring emphasis on transformation geometry in mathematics education. The contextual perspective focuses on the current state of geometry teaching and learning in Ugandan secondary schools, highlighting curriculum priorities and persistent challenges.

The theoretical perspective introduces the Van Hiele model of geometric reasoning, together with the constructivist learning theory, which underpins the study's instructional approach and analysis. Finally, the conceptual perspective outlines the key constructs, geometry attitudes, instructional strategies, and Van Hiele levels, and explains their interrelationships.

1.1.1 Historical Perspective

Geometry is the science of space and shapes that provides the basis for various practical applications across different disciplines, which include, building and construction, and industrial designs. It holds massive importance as a foundational branch of mathematics; its study fosters critical thinking, problem-solving skills, and spatial reasoning abilities essential for solving complex problems in varied disciplines (Asli & Zsoldos-Marchis, 2021; Sharma, 2019). According to Kivkovich and Chis (2016), understanding geometry principles is necessary to interpret the physical world and contribute to technological advancements.

Due to its relevance, geometry is part of the curriculum at all education levels, from pre-primary (nursery) up to university level. Despite its widespread use across educational levels, learners struggle with geometry concepts and applications, leading to fear and anxiety about the subject (Zeinul & Silmi, 2022).

Accordingly, this impacts the overall learners' academic performance in mathematics. The problem of low performance in geometry has been documented in several countries, including Indonesia, South Africa, Nigeria, Pakistan, Sri Lanka, and Italy (Ayebale et al., 2020; Ubi et al., 2018; Zeinul & Silmi, 2022). Given the widespread nature of this problem, Ayebale et al. (2020) recommend an urgent global intervention to mitigate the problem and improve geometry education outcomes.

Consistent with this, Ngirishi and Bansilal (2019) note that one of the most common difficulties learners face is understanding basic geometry terminology and interpreting the properties of shapes. They further observe that most learners function at lower levels of geometric thinking, limiting their capacity to comprehend more advanced geometry concepts. The primary challenge is how concepts are delivered to learners in geometry classrooms. (Kivkovich & Chis, 2016). This often relies on traditional approaches, which, according to Ardeleanu (2019), may not cater to diverse learning needs, resulting in low learner engagement and a limited understanding of geometry concepts.

Misconceptions and negative attitudes toward geometry among learners further escalate the challenge. They struggle with vocabulary, application of formulas, and theorems, and perceive geometry as a problematic and uninteresting subject (Kivkovich & Chis, 2016). In addition, the usage of mathematical instruments like protractors and compasses still challenges many learners, resulting to skipped questions and increasing difficulties (Luneta, 2015).

Generally, there is a need to improve geometry education, focusing on enhancing conceptual understanding and utilizing effective instructional strategies that engage learners during the process of learning. According to Moru et al. (2021), addressing this challenge will make it easy for educators to provide appropriate assistance to the students. In turn they will be able to develop a strong understanding of geometry concepts, thereby improving their overall proficiency in mathematics.

The prior findings discussed above reveal that many learners across the globe lack conceptual knowledge in geometry, which affects learners' ability to respond correctly to geometry questions. Likewise, learners in Ugandan secondary schools face similar challenges. , some of which are discussed in the next subsection.

Thus, the current study explores these challenges by investigating the effectiveness of key instructional strategies for teaching transformation geometry, a critical domain in geometry because it demands learners to integrate concepts of congruence, symmetry, and spatial reasoning while transitioning from concrete visualization to abstract thought. Mastery of transformation geometry is widely regarded as foundational for broader geometric understanding and advanced mathematical problem-solving (Ndungo, 2024).

While challenges in geometry education are evident across many education systems worldwide, these issues manifest uniquely within the Ugandan context due to curriculum demands, resource constraints, and variability in teacher preparedness. National examination reports have consistently highlighted low performance in transformation geometry, particularly in tasks requiring higher-order

reasoning. This local reality, when viewed against the backdrop of global concerns, underscores the urgent need for pedagogical approaches that are both contextually relevant and theoretically grounded.

1.1.2 Contextual Perspective

Uganda's lower secondary school mathematics curriculum identifies geometry as a key area that equips learners with essential geometric skills, such as shape construction, measurement, reflection, rotation, and enlargement (NCDC, 2019a). However, learners are facing numerous challenges in learning geometry due to the nature of the instructional practices that have been predominantly teacher-centered, relying on traditional methods like demonstrations on the blackboard, which limit students level of conceptual understanding, leading to rote memorization of concepts and contributing to persistent challenges in geometry education (Brijlall & Abakah, 2022).

Actual classroom conditions shed further light on these challenges. Teacher preparation in geometry instruction often varies significantly across schools, with many educators lacking specialized training in both content knowledge and pedagogical strategies suited for geometry (Kidega et al., 2023). This shortfall limits their ability to effectively facilitate learners' conceptual understanding and to employ instructional approaches that cater for diverse learning needs.

Furthermore, the provision and utilization of teaching and learning resources pose a significant challenge. Numerous schools, particularly those in rural settings, experience shortages of vital materials, including manipulatives, geometric instruments (such as protractors and mirrors), and access to technology-based tools like GeoGebra (Bashiru & Umar, 2018). Where materials exist, limited teacher familiarity with innovative tools curtails their effective integration into lessons.

Furthermore, large class sizes, insufficient time allocated to mathematics instruction, and inconsistent use of formative assessments contribute to learners' difficulties (Mangarin & O.Caballes, 2024). These factors combine to create an environment where rote learning predominates over active

engagement and exploration, thereby constraining the development of higher-order geometric reasoning skills.

Ndungo et al. (2024) analyzed the Uganda Certificate of Education (UCE) mathematics reports on candidates' performance from 2016 to 2022[1]. Their analysis revealed persistent challenges learners encounter in geometry-related assessments, including difficulties with spatial visualization, integrating algebraic manipulation with geometric concepts, applying geometric principles to problem-solving tasks, and relating theoretical knowledge to real-world contexts. Additionally, the analysis showed that geometry consistently exhibits lower performance and attempt rates relative to other mathematics topics such as matrices, statistics, sets, business calculations, and many others. These results align with the global trends in geometry education discussed in Subsection 1.1.1, emphasizing that geometry remains a predominantly challenging area of mathematics for many learners.

Similar assessments have revealed comparable results. For example, NAPE report for 2014 indicates a significant proficiency gap, with only 42.8% of boys and 38.9% of girls rated proficient in mathematics, with geometry, Cartesian coordinates, and transformation identified as particularly challenging areas (UNEB, 2014). This issue has persisted, as evidenced by subsequent NAPE reports from 2017 to 2021 (UNEB, 2017a, 2021a).

UNEB attributes these challenges to the way geometry is taught and has put forward several recommendations to mitigate these challenges, such as integrating technology into the teaching of geometry, using learner-centered methods of teaching, providing numerous practice activities to learners, and making the teaching of geometry hands-on (UNEB, 2023; Ndungo et al., 2024). The above recommendations highlight the role of modifying instructional strategies to ensure learners are engaged,

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[1] Ndungo, I., Akugizibwe, E., & Balimuttajjo, S. (2024). Analyzing trends and suggested instructional strategies for geometry education: Insights from Uganda Certificate of Education-Mathematics Examinations, 2016–2022. *African Journal of Teacher Education, 13*(2), 153–186. https://journal.lib.uoguelph.ca/index.php/ajote/article/view/8106

integrating technology effectively, and providing regular activities. However, the questions that remain unanswered are the specific instructional strategies and technologies that are suitable for geometry instruction.

The Van Hiele Phased instruction, a subject-specific strategy for geometry instruction, sheds light on addressing the above questions. The strategy has been reported to effectively support learners' understanding of geometry (Chen, 2009; Machisi & Feza, 2021; Wahab et al., 2016). However, its effectiveness in Uganda remains underexplored, as most studies and evidence are from other countries, including Nigeria, Pakistan, Sri Lanka, South Africa, Indonesia, Italy, and Malaysia (Ubi et al., 2018; Ayebale et al., 2020; Zeinul & Silmi, 2022). Also, given the technological advancements in the education landscape, call for technology integration in geometry instruction and mathematics in general.

Recent studies have indicated that incorporating technology like GeoGebra, Desmos, YouTube, Geometer's Sketchpad, etc, into the teaching of geometry supports visualization, engagement, and conceptual understanding (Alkhateeb & Al-Duwairi, 2019; Chisenga & Alfred, 2021; Chivai et al., 2022; Dahal et al., 2022; Pavethira & Leong, 2017; Uwurukundo et al., 2021).

This study aims to assess the effectiveness of TVHPI and CVHPI in enhancing learners' geometry proficiency and how these strategies inform learners' experiences and attitudes towards geometry, providing empirical insights that can inform evidence-based geometry education and advance pedagogical practices.

1.1.3 Theoretical Perspective

This study utilized constructivist learning theory, in conjunction with the Van Hiele theory of geometric reasoning. The constructivist learning theory views learning as an active process that allows learners to engage and interact with the milieu and construct new knowledge or understanding built on their experiences or existing knowledge (Bada & Olusegun, 2015). The instructional strategies investigated in

the current study have their roots in the principle of active involvement in learning by learners, and the social interactions and individual differences, making constructivist learning theory informative to the study and the instructional activities.

The Van Hiele theory of geometric reasoning classifies learners into different levels of geometric reasoning, depending on their ability to perform different geometry tasks. The levels are visualization, analysis, abstraction (informal deduction), (formal) deduction, and lastly, rigor (Crowley, 1987; Vojkuvkova, 2012). The theory aids the understanding of how learners cognitively develop their reasoning in geometry and provides a structured lens for assessing instructional strategies (this is detailed in Subsection 2.1.2). The current study aims to establish the interplay among instructional strategies, learners' attitudes, experiences, and progression through Van Hiele levels of geometric reasoning, focusing on Van Hiele's phased instruction.

The Van Hiele Phased instruction proposed in the Van Hiele theory of geometry reasoning as an instruction strategy (explained in detail under 1.1.4.3) has been found from prior research to be effective in supporting learners' proficiency and progression in geometry (Ana & Ana, 2016; Suwito et al., 2016; Kambilombilo & Sakala, 2015; Mahlaba & Mudaly, 2022; Ngsirishi & Bansilal, 2019; Zhou et al., 2022; Wulandari et al., 2021). This implies that the Van Hiele Phased instruction has demonstrated success in supporting of geometry instruction in similar contexts, laying the groundwork for further investigation within the specific limits of the current study.

1.1.4 Conceptual Perspective

The variables examined in this study are related to instructional strategies and geometry education as derived from the established framework. The interrelationships and impact are explored to provide evidence on their role in supporting the improvement of geometry education in Uganda.

1.1.4.1 Geometry Attitudes and Perceptions

Geometry attitudes measure learners' feelings, beliefs, and emotional characters towards geometry (Pavlovicova & Zahorska, 2015). On the other hand, perceptions measure how learners interpret the usefulness, the difficulty, and the teaching strategies of geometry. Attitudes are generally classified into affective, behavioral, and cognitive attitudes.

Affective attitudes relate to students' interest, confidence, and emotional responses toward geometry, while behavioral attitudes reflect their participation, effort, and engagement in learning activities, and cognitive attitudes encompass their perceptions of the relevance and applicability of geometry in real-world contexts (Smith & Jones, 2020). The current study investigated the mediating role of geometry attitudes in the relationship between instructional strategies and learners' progression through Van Hiele levels, to understand how positive geometry attitudes facilitate deeper engagement and understanding of geometry concepts.

1.1.4.2 Van Hiele Levels of Geometric Reasoning

The Van Hiele levels of geometric reasoning represent progressive stages of cognitive development that learners can attain through exposure to diverse geometric environments and concepts. There are visualization, analysis, abstraction, formal deduction, and rigor (Vojkuvkova, 2012).

They represent the outcome variable of the current study, reflecting the learners' ability to understand and apply geometry concepts at different cognitive stages (Sert Celik & Kaleli Yilmaz, 2022). Figure 1 shows the five Van Hiele levels and their distinct properties, while Figure 25, located in the appendix section, shows the activities under each level specific to transformation geometry in S.3.

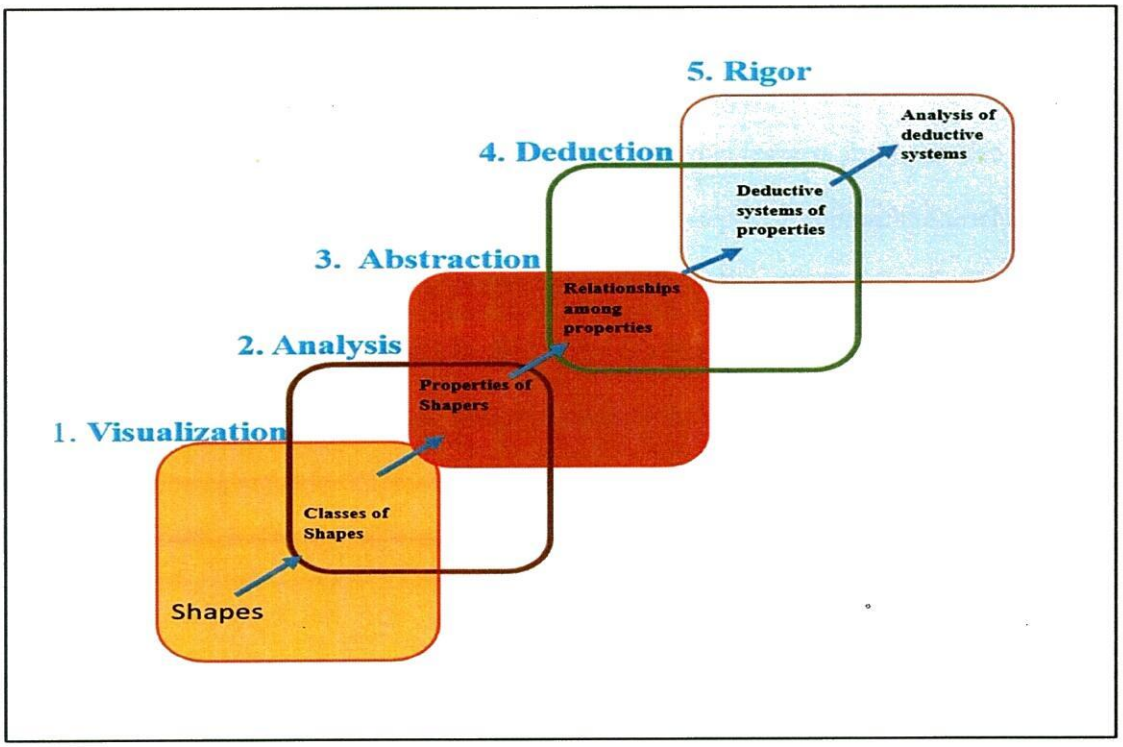


Figure 1: Showing the Van Hiele levels of geometric reasoning and their distinct properties

1.1.4.3 Instructional Strategies

An instructional strategy is an organized blueprint formulated to achieve specific learning outcomes by selecting and sequencing various teaching methods, techniques, and resources to engage learners effectively. As distinguished from broader educational philosophies (teaching approaches), specific frameworks (teaching methods), and concrete actions within those frameworks (teaching techniques), an instructional strategy includes a comprehensive approach to instruction (Thapa et al., 2016).

This study employed the conventional Van Hiele phased instruction in comparison with the Technology-enhanced Van Hiele phased instruction, positioned within the competency-based curriculum framework. The CVHPI and TVHPI serve as dimensions of instructional strategies; they are independent variables and are presumed to impact learners' attitudes towards geometry and, consequently, their progression through Van Hiele levels of geometric reasoning.

Conventional Van Hiele Phased Instructional (CVHPI)

This instructional strategy resonates from the Van Hiele theory of geometric reasoning, tailors activities to the different Van Hiele levels, and focuses on moving learners from one Van Hiele level to the other (Abdullah & Zakaria, 2013b). This strategy follows a step-wise approach of teaching that cycles through five phases: information/inquiry phase, directed orientation, explication, free orientation, and integration. This means the strategy uses inquiry-based teaching, engages the learners in learning, and learners cycle through the different stages until they master the concepts of a particular level of geometric reasoning, following an inductive approach, and guided by the teacher before they can proceed to the next level (Crowley, 1987; Vojkuvkova, 2012). It offers a domain-specific framework for developing geometric reasoning that differs from general cognitive models such as Piaget's theory or Bloom's taxonomy.

Unlike Piaget, the Van Hiele levels are not strictly age-dependent, and unlike Bloom, they are specifically tailored to geometry, emphasizing how students transition from visual recognition to deductive reasoning (Alanazi, 2016). Compared to Vygotsky's constructivist theory, the Van Hiele model provides a more clearly sequenced instructional pathway that is particularly helpful for teachers designing structured lessons. For secondary school teachers, VHPI is uniquely practical. Its well-defined levels and instructional phases serve as a diagnostic tool and an instructional guide, helping teachers scaffold geometric understanding to respect each learner's current reasoning level. The five phases of the Van Hiele Phased Instruction (VHPI) are depicted in Figure 2.

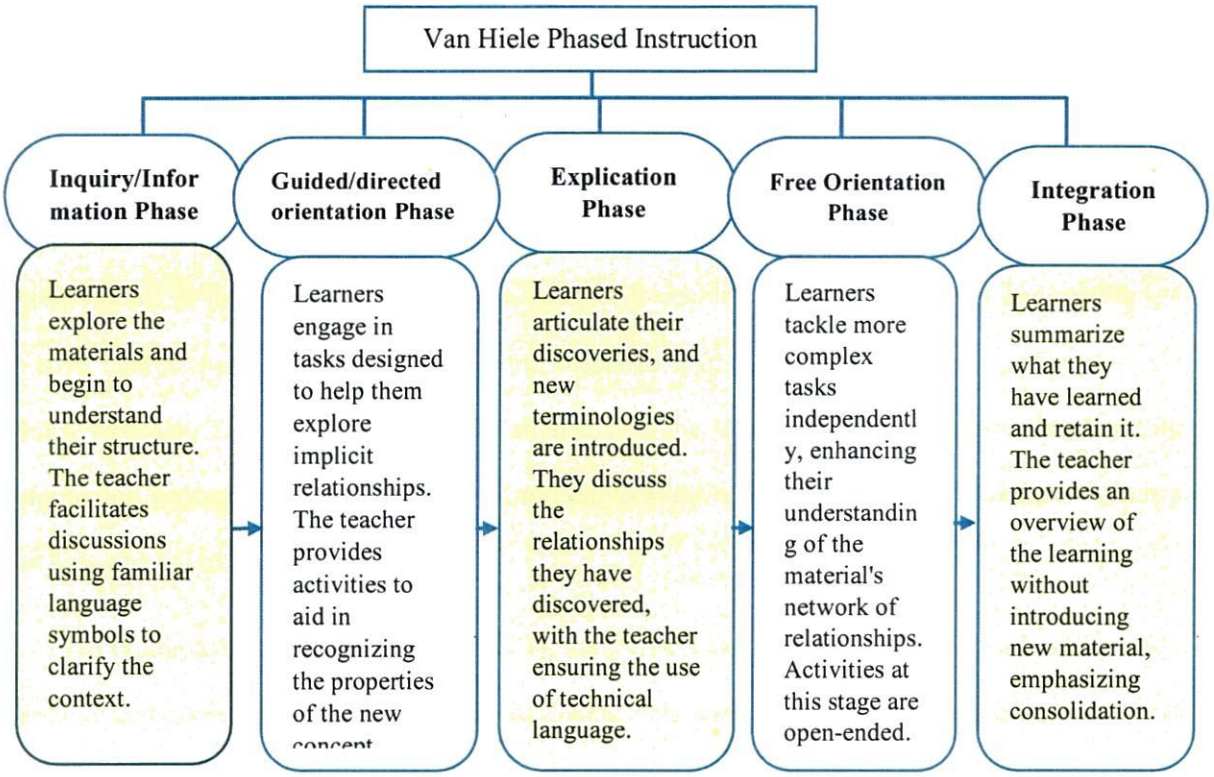


Figure 2: The five phases of the Van Hiele phased Instruction.

Figure 2 illustrates the five phases of Van Hiele phased instruction: (1) Information/inquiry, where learners explore concepts; (2) Directed Orientation, focusing on exploration activities; (3) Explication, where learners articulate understanding and verbalize the generated knowledge; (4) Free Orientation, applying knowledge to complex problems; and (5) Integration, consolidating concepts for deeper comprehension.

Technology-enhanced Van Hiele Phased Instructional (TVHPI)

Technology-enhanced Van Hiele Phased Instruction refers to implementing the Van Hiele model of geometric reasoning by integrating digital technology into each of its five instructional phases. This approach retains the core theoretical structure of the Van Hiele model but leverages technological tools to

facilitate dynamic visualization, interactive manipulation, and real-time feedback during geometry learning.

This study employed GeoGebra, a dynamic geometry software, as the instructional tool for implementing TVHPI. GeoGebra was developed by Markus Hohenwarter in 2001 when he was a graduate student at the Salzburg University in Austria. It enables learners to construct and manipulate geometric figures, perform transformations, and visualize mathematical relationships with immediate graphical feedback. Through structured tasks aligned with the Van Hiele phases, learners interact with transformation geometry concepts in a visual and exploratory manner, guided by the teacher within a technology-rich environment.

According to Chisenga and Alfred (2021), the learners can draw shapes, plot coordinates, rotate, and enlarge geometric shapes with the use of GeoGebra. This instructional strategy is implemented using computers/laptops or smartphones as teaching and learning tools and for visualizing geometry concepts, including 3-D shapes (Tamam & Dasari, 2021). For example, in the visualization phase, learners can use GeoGebra to manipulate geometric figures and see the effects of transformations in real time.

Mosese and Ogbonnaya (2021) claim that incorporating technology like GeoGebra into Van Hiele phase-based instruction can make the learning process more interactive, visual, and adaptable to learners' evolving levels of geometric reasoning, providing a more engaging and effective way to teach geometry. Figure 3 shows the interface of GeoGebra software.

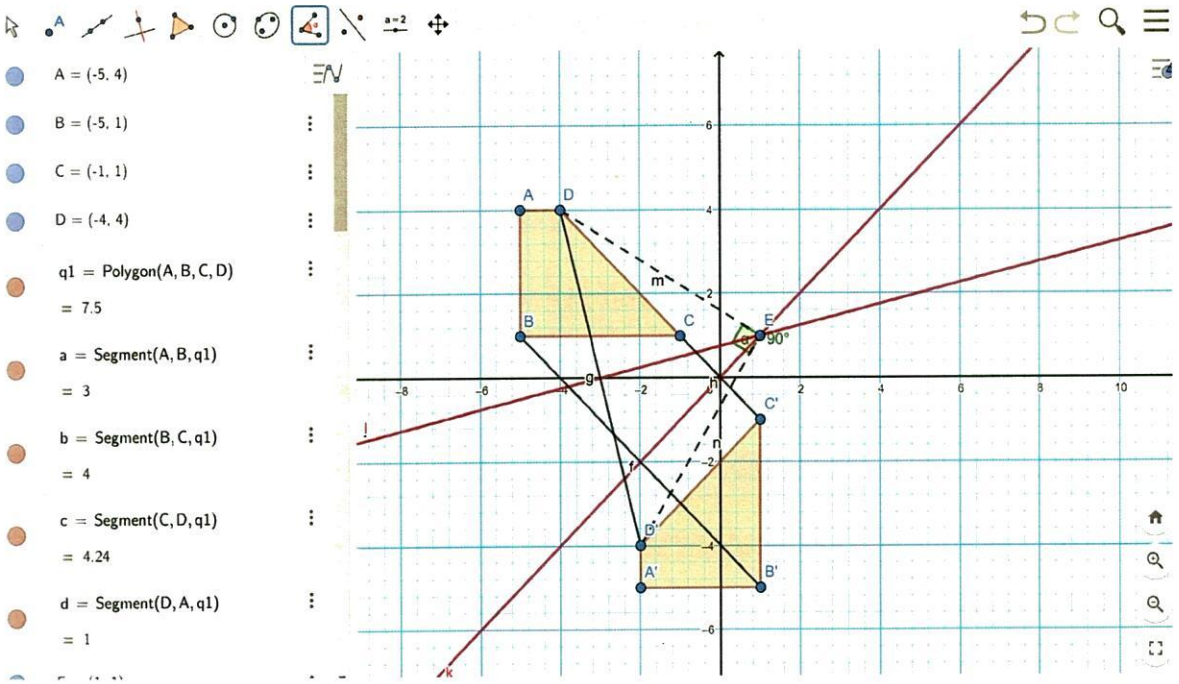


Figure 3: *GeoGebra environment showing how to find the angle of rotation of a shape.*

Figure 3 provides a complete preview of the GeoGebra environment, which is central to the Technology-Enhanced Van Hiele Phased Instruction strategy. The interface comprises three main components.

Graphics View (center): This interactive space visualizes geometric objects and transformations. In the displayed example, a quadrilateral ABCD undergoes a rotation, enabling learners to dynamically explore the effects of transformations on shape, size, and position.

Algebra View (left panel): Lists coordinates of points, equations of lines, and properties of constructed objects, establishing a direct link between algebraic and geometric representations. This supports deeper conceptual understanding and helps learners verify their results numerically.

Toolbar (top): Provides essential tools for constructing points, polygons, axes, and transformations such as rotation, reflection, and translation. The toolbar enhances flexibility, allowing learners to engage with the material through exploration and experimentation.

GeoGebra's dynamic and multi-representational nature supports learners' progression through Van Hiele levels by encouraging visualization, directed orientation, and deductive reasoning. Learners can test conjectures, manipulate figures in real time, and instantly observe changes, making abstract concepts more tangible.

1.1.4.4 Implementation Differences between CVHPI and TVHPI

The Van Hiele Phased Instruction model comprises five instructional phases: Information, Guided Orientation, Explication, Free Orientation, and Integration. Each phase is intended to progressively support learners as they develop increasingly sophisticated levels of geometric reasoning. In this study, both CVHPI and TVHPI followed this structured sequence, with their key difference being the mode of instructional delivery, traditional tools versus digital technology.

1. Information Phase

The Information Phase activates learners' prior knowledge and connects new learning to familiar, real-world contexts. It introduces essential vocabulary, symbols, and foundational ideas. In CVHPI, the teacher prepares and uses conventional resources such as graph paper, mirrors, rulers, and chalkboards to present the basic concepts of transformation geometry. Real-life examples and physical manipulatives help learners recognize and describe real-life geometry transformations.

In TVHPI, the identical conceptual content is introduced using digital tools. Teachers use computers or tablets with GeoGebra to present transformation geometry concepts, allowing learners to visualize and interact with geometric objects in a virtual environment.

2. Guided Orientation Phase

In the Guided Orientation Phase, learners participate in structured activities to assist them discover properties and relationships through carefully sequenced activities. In CVHPI, the teacher guides students through manual exercises using tracing paper, graphing tools, and physical models. Students follow step-by-step instructions to perform transformations, including reflections, rotations, and translations.

The focus is on observation, measurement, and identification of invariant properties. In TVHPI, learners carry out similar tasks using GeoGebra. They manipulate digital objects by dragging points and adjusting sliders to observe transformations dynamically. The teacher facilitates this interaction by directing attention to specific features, helping learners recognize patterns and geometric properties within the digital environment.

3. Explication Phase

The Explication Phase emphasizes student articulation or verbalizing their understanding. Learners are encouraged to explain orally the properties and relationships they have observed during guided activities. In CVHPI, learners discuss their findings orally or in writing, referencing their manually drawn figures to explain the effects of transformations. Teachers prompt students to explain terms such as line of reflection, angle of rotation, or center of enlargement, helping them move from informal to formal vocabulary.

In TVHPI, students similarly express their understanding using observations from GeoGebra activities. They may explain screenshots, participate in class discussions, or demonstrate their findings directly within the software. The teacher facilitates the transition from intuitive reasoning to structured geometric language.

4. Free Orientation Phase

Learners engage in more complex and less structured tasks in the Free Orientation Phase. These activities require applying learned concepts to solve problems that may have multiple solutions or varied approaches.

In CVHPI, students solve geometric tasks by hand, using traditional tools to perform combined transformations or explore unfamiliar configurations. These tasks require critical thinking, and learners are encouraged to justify their methods and verify results through construction and measurement.

In TVHPI, students use GeoGebra to experiment with different transformation sequences, test conjectures, and explore "what-if" scenarios. The digital platform allows for rapid iteration, enabling learners to analyze the effects of changes and refine their understanding. Both methods provide opportunities for problem-solving and conceptual reinforcement, though the modes of interaction differ.

5. Integration Phase

The Integration Phase consolidates learning by helping learners reflect on the key ideas and strategies developed in earlier phases. In CVHPI, this may include written summaries, presentations of hand-drawn transformation maps, or teacher-led reviews of concepts. Assessment tasks evaluate procedural skills and conceptual understanding based on students' written or drawn responses. In TVHPI, learners demonstrate understanding by completing digital tasks in GeoGebra, explaining their transformation steps, or interpreting visual outputs. Teachers assess both procedural fluency and reasoning through screen-based demonstration.

1.1.4.5 Role of the Teacher and Learner Engagement

In CVHPI, the teacher typically provides more direct instruction, ensuring learners understand the mechanics of transformation using physical tools. Interaction is often structured, with the teacher guiding learners closely through each step.

In TVHPI, the teacher adopts a more facilitative role, supporting learners as they explore and interact with digital tools. Learners often have greater control over the pace and depth of exploration, while the teacher monitors understanding and offers individual guidance as needed. The teacher supports learners' progression through the Van Hiele levels in both approaches by aligning tasks with the instructional phases.

1.1.4.6 Positioning Within the Study

In this study, CVHPI and TVHPI align with the Van Hiele instructional model to ensure consistency in content, sequencing, and pedagogical intent. The variation lies solely in the medium of instruction, conventional tools versus dynamic digital environments.

By maintaining consistent instructional phases across both groups, the study enables a fair and structured comparison of how the different delivery modes may inform the experience and understanding of the learners.

1.1.4.7 Conceptual Framework

This study adopts Rosenberg's Tripartite Model of Attitude (1980) and Van Hiele's Model of Geometric Thought (1957) to examine learners' attitudes, experiences, and understanding of transformation geometry.

While the Van Hiele Model provides a theoretical lens for understanding learners' cognitive progression in geometric reasoning, this study also draws on the Tripartite Model of Attitude to address the affective domain, which is critical in shaping learning behaviors and engagement. The Tripartite Model conceptualizes attitude as comprising three interrelated components: cognitive (beliefs about geometry), affective (feelings toward geometry), and behavioral (tendencies to act in certain ways).

Integrating these models ensures a holistic approach, capturing both cognitive progression (Van Hiele levels) and attitudinal dispositions that influence or interact with this progression. For instance, while the Van Hiele model predicts that learners advance through hierarchical reasoning levels when instruction is well-structured, the Tripartite Model helps explain why learners may resist or embrace such progression, depending on their beliefs, emotions, and behaviors toward geometry. In this study, these frameworks intersect methodologically:

Van Hiele Model anchors the assessment of learners' geometric reasoning (pre-test and post-test VHL), while the tripartite Model informs the measurement and interpretation of geometry attitudes through the TGAS scale, enabling the analysis of whether attitudes predict or mediate learners' movement across Van Hiele levels.

This interplay addresses a critical gap: while previous studies often examine either reasoning or attitude in isolation, this research investigates their combined influence within different instructional contexts (CVHPI vs. TVHPI), offering deeper insights into how cognitive and affective factors jointly shape geometry learning.

Figure 4 visually represents the interconnections and relationships between the variables under consideration in the study.

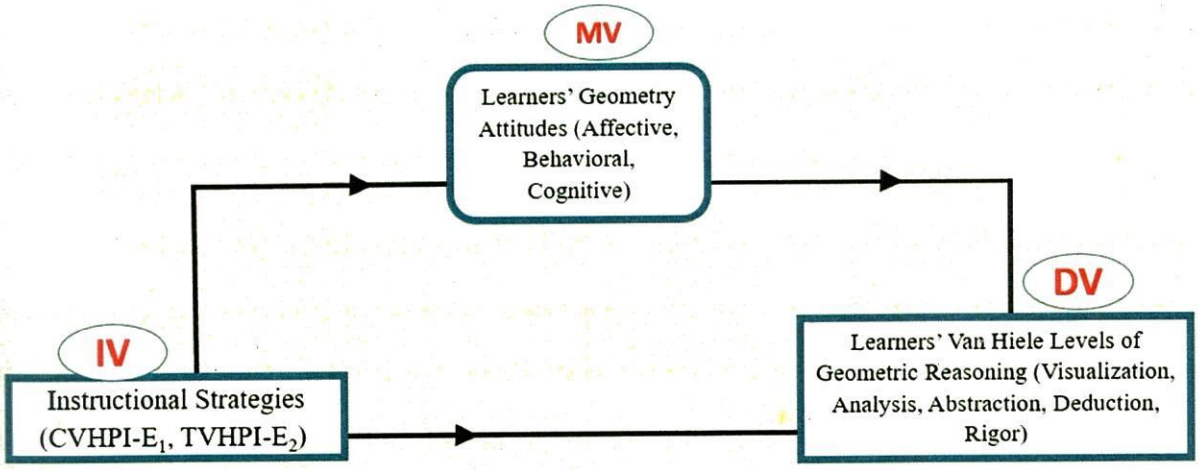


Figure 4: Conceptual Framework showing the relationship between the instructional strategies, learners' geometry attitudes, and Van Hiele's geometric reasoning.

***Source**: Adopted from* Rosenberg (1980) *and Van Hiele's Theory of geometric Reasoning* (Van Hiele, 1957).

Note: IV = Independent Variable, MV =Moderating Variable,

DV =Dependent Variable, E = Experiment

According to Figure 4, the tripartite model categorizes attitudes into cognitive (beliefs and perceptions), affective (emotions and motivation), and behavioral (engagement and persistence), enabling a comparative analysis of CVHPI and TVHPI in shaping learners' attitudes (Objective 1). The Van Hiele Model provides a framework for evaluating students' progression through Van Hiele reasoning levels under different instructional strategies (Objective 2), utilizing the first four levels and excluding the last level, as it is outside the scope and context of the current study.

Furthermore, it allows for an inquiry into the mediating role of attitudes on the relationship between instructional strategies and learners' progression (Objective 3). Finally, by incorporating learners' experiences and understanding (Objective 4), the study provides a comprehensive evaluation of how instructional strategies shape learners' experience, engagement, and understanding of transformation geometry, pinpointing instructional-related challenges and coping mechanisms.

The Rigor level, which represents the highest Van Hiele level, was included in the conceptual framework to maintain theoretical completeness and reflect the full structure of Van Hiele's model. However, this study did not empirically assess Rigor because it involves advanced axiomatic reasoning and abstract proof structures, which extend beyond the learning expectations of Senior Three learners under Uganda's Competency-Based Curriculum (CBC). This level represents an advanced stage of formal deductive reasoning, typically involving abstract axiomatic systems and logical proofs, often achieved in advanced mathematics education beyond the lower secondary level. The study focused on the first four levels, Visualization, Analysis, Informal Deduction, and Deduction, because these align closely with the competencies emphasized in Uganda's Competency-Based Curriculum (CBC) at the secondary school level. While Rigor is important in developing mathematical maturity, it generally falls outside the expected learning outcomes for Senior Three learners under the current curriculum framework.

Building on the relationships illustrated in the conceptual framework, the present study operationalizes these linkages through a mixed-methods, quasi-experimental design. The framework's key constructs, instructional strategies, learners' geometry attitudes, and Van Hiele levels of geometric reasoning are examined using both quantitative and qualitative approaches.

Two instructional strategies, Conventional Van Hiele Phased Instruction (CVHPI) and Technology-Enhanced Van Hiele Phased Instruction (TVHPI) with GeoGebra, are implemented among Senior Three learners in rural and urban secondary schools. Quantitative measures include pre- and post-tests assessing attitudes and geometric reasoning, while qualitative interviews capture learners' perspectives and experiences.

This approach ensures that the interactions and effects depicted in the framework are systematically investigated, providing a comprehensive understanding of the interplay among the study variables.

1.2 Statement of the Problem

Geometry is a foundational area of mathematics education, promoting problem solving, spatial reasoning and logical thinking skills essential for STEM disciplines (Jones, 2002; Duval, 1998). Transformation geometry, in particular, is critical because it links algebraic reasoning with geometric visualization and has abundant real-life applications (Brijlall & Abakah, 2022).

Despite its importance, studies indicate that learners worldwide often experience difficulty mastering this area, which is frequently attributed to traditional instruction that emphasizes procedures over conceptual understanding (Battista, 2001; Clements & Sarama, 2021)

Globally, the Van Hiele model has been widely used to explain learners' progression in geometric reasoning (Alex & Mammen, 2016; Larbi, 2021). However, limited research has examined how different

instructional strategies, especially conventional versus technology-enhanced approaches, affect both attitudes and progression through the Van Hiele levels. Moreover, the role of attitudes as predictors or mediators of conceptual growth remains underexplored, leaving a gap in understanding how cognitive and affective dimensions interact to support geometry learning.

In Uganda, transformation geometry remains a persistent area of weakness, as shown in national assessments (Kiwanuka et al., 2015; Ndungo et al., 2024). Secondary classrooms are largely teacher-centered, with minimal use of dynamic teaching tools or strategies that actively engage learners (Kidega et al., 2023). Although the Competency-Based Curriculum (CBC) emphasizes learner-centered pedagogy and ICT integration, there is no empirical study examining how CVHPI and TVHPI influence learners' attitudes and progression through Van Hiele levels in the Ugandan context. This absence of local evidence limits data-driven curriculum implementation and teacher professional development.

To address these gaps, a mixed-methods design was adopted, a quasi-experimental design that combines quantitative measures of learners' attitudes and progression through the Van Hiele levels with qualitative insights into their learning experiences, enabling a comprehensive examination of the interplay among instructional strategies, attitudes, and geometric reasoning.

1.3 Purpose of the Study

The study assessed the interplay among Technology-Enhanced and Conventional Van Hiele Phased Instruction, learners' attitudes towards geometry, and their progression through Van Hiele levels of geometric reasoning.

1.4 Specific Objectives, Hypotheses, and Research Questions

1.4. 1 Specific Objectives

Four specific objectives guided the study:-

1. To compare the effect of CVHPI and TVHPI on shifts in learners' attitudes toward transformation geometry.
2. To compare the effect of instructional strategies on learners' progression through Van Hiele's levels
3. To examine the predictive and mediating roles of geometry attitudes in learners' Van Hiele levels across the two instructional strategies.
4. To examine learners' understanding and experiences in learning transformation geometry using TVHPI and CVHPI.

1.4. 2 Hypotheses

Each of the first, second, and third objectives had two hypotheses, as listed below

1. **H_{01}:** Learners exposed to TVHPI or CVHPI will not exhibit statistically significant shifts in attitudes toward transformation geometry from pre-test to post-test.
2. **H_{02}**: There is no significant difference in attitude shifts between learners taught using TVHPI and those taught using CVHPI.
3. **H_{03}**: Learners exposed to TVHPI or CVHPI will not exhibit statistically significant progression in Van Hiele levels from pre-test to post-test.
4. **H_{04}**: There is no significant difference in Van Hiele levels progression between learners taught using TVHPI and those taught using CVHPI.
5. **H_{05}**: Geometry attitudes do not significantly predict learners' Van Hiele levels

6. H_{06}: Attitude change does not mediate the relationship between instructional strategy and learners' progression through Van Hiele levels.

1.4. 3 Research Questions

Objective four was achieved via two research questions, as listed below

1. **RQ_1:** What challenges and support needs are associated with TVHPI and CVHPI?
7. **RQ_2:** What are learners' perceptions of the effectiveness of CVHPI and TVHPI in enhancing their understanding of transformation geometry?

1.5 Justification of the Study

The study offers practical evidence concerning the efficacy of different instructional strategies explicitly embedded in Van Hiele's theory of geometric reasoning, supporting geometry education in Uganda. The study offers concrete insights that can inform educational policy and practice and facilitate informed decisions that can positively impact the geometry education landscape in Uganda.

Furthermore, the study is situated within the global movement toward leveraging technology to enhance learning experiences. It recognizes the importance of integrating technology into geometry instruction and offers evidence on how technology interacts with other educational factors, including attitudes and cognitive development.

The results will inform pre-service teacher training by highlighting the need to equip future teachers with skills in dynamic geometry software and conceptual instruction, and guide in-service professional development programs to support teachers in adopting blended, ICT-supported pedagogies. Ultimately, the study contributes to enhancing mathematics teaching practices, fostering positive learner attitudes, and promoting sustainable instructional improvement in Ugandan secondary schools.

1.6 Scope of the Study

1.6.1 Content Scope

This study focused on the role of instructional strategies in shaping learners' attitudes toward transformation geometry and influencing their progression through the Van Hiele levels of geometric reasoning. The study investigated whether these strategies could boost learners' conceptual understanding of transformation geometry. In addition to measuring attitudinal change and Van Hiele level progression, the study explored the challenges learners encountered during the instructional process, the types of support they required, and their perceptions of the effectiveness of each instructional approach.

To determine the effectiveness of CVHPI and TVHPI, a mixed-methods quasi-experimental design was employed. Quantitatively, learners' attitudes were assessed using the Transformation Geometry Attitude Scale (TGAS), while their conceptual progression was measured using the Van Hiele Geometry Achievement Test (VGAT). Inferential analyses, including the Mann–Whitney U Test, the Difference-in-Differences models, the Wilcoxon Signed Rank Test, and Generalized Ordered Logit models, were applied to test the research hypotheses. Qualitatively, classroom observations and learner interviews provided contextual insights into engagement and conceptual understanding.

1.6.2 Geographical Scope

The research was conducted in six secondary schools located in western Uganda with the Rwenzori sub-region. These schools were purposively selected to ensure equitable representation from both urban and rural settings, with three schools based in Fort Portal City (an urban setting) and three in Kasese District (a rural setting). This geographical distribution allowed the study to capture diverse learner experiences influenced by differing access to technological infrastructure, school resources, and instructional environments.

The region was also selected due to its strategic relevance to ongoing science and mathematics education reforms, its position relative to other regions regarding learners' achievement levels in mathematics based on national assessments, and the researcher's familiarity with the educational context.

1.6.3 Time Scope

The study spanned from April 2024 to May 2025. This thirteen-month period encompassed all major research activities, including a literature review, developing and validating research instruments, administering pre-tests, implementing instructional strategies (CVHPI and TVHPI), administering post-tests, and collecting qualitative data through interviews.

The timeframe also allowed for adequate monitoring of learner engagement during instruction and the iterative refinement of tools and techniques used in data analysis. Finally, this period included preparing preliminary findings for presentation and dissemination to stakeholders.

1.7 Definition of Terms

1.7.1 A Learner/Student

In this thesis, the term student refers to an individual formally enrolled in a secondary school program (e.g., Senior Three classes). The term learner is used when emphasizing the process of acquiring knowledge, skills, or attitudes, particularly within the act of learning or in broader pedagogical discussions. Both terms appear in this study, with usage determined by the context of either formal enrollment or active engagement in learning.

1.7.2 Geometry

Geometry is among the major topics of mathematics, covering shapes, sizes, shape properties and space, and the relationships between various geometric objects. It explores points, lines, angles, surfaces, transformations, and solids and their interactions in two-and three-dimensional space.

1.7.3 Transformation Geometry

Transformation geometry is a section of geometry that focuses on studying how geometric shapes, lines, and points undergo operations. These operations, called transformations, can be modified while preserving their fundamental properties. These transformations can alter the objects' position, size, orientation, or shape.

Depending on the type of transformation, some properties (such as distances, angles, and parallelism) may be preserved, while others (such as size or shape) may be altered. Common transformations include translations, rotations, reflections, and dilations, each affecting the object differently while maintaining specific properties.

1.7.4 Attitude

Geometry attitudes refer to learners' beliefs and emotional dispositions toward geometry learning. These attitudes influence educational experiences, teaching approaches, and individual dispositions toward mathematics. Geometry attitudes are foundational in this study, mediating between instructional strategies and learners' progression through Van Hiele levels. Positive geometry attitudes are hypothesized to facilitate deeper engagement and understanding of geometric concepts.

1.7.5 GeoGebra

GeoGebra is a dynamic mathematics software program used in instruction to facilitate the teaching and learning of various mathematical concepts, including geometry, algebra, calculus, and more (Dahal et al., 2022). It provides a wide range of mathematical tools and resources for educators and learners, allowing them to explore and visualize mathematical ideas, create interactive diagrams, and solve mathematical problems.

1.7.6 Learning experiences

These refer to how students interact with and perceive the learning process while learning geometry. This includes their emotional responses, such as motivation or frustration, their cognitive engagement with geometric concepts, their overall perception of the effectiveness and accessibility of the instructional strategy, and how these experiences shape their understanding of geometry and influence their attitudes toward the subject.

1.7.7 Conceptual and Procedural Understanding

Conceptual understanding is grasping geometry ideas, recognizing patterns, and understanding relationships between concepts. In contrast, procedural understanding refers to the ability to follow specific steps, algorithms, or rules to solve mathematical problems accurately (Salim & Gilar, 2020).

1.8 Organization of the Thesis

This thesis is structured into five chapters, beginning with Chapter 1, which introduces the study, and then Chapter 2, which situates the study within existing research and reviews the related literature.

Next, Chapter 3 details the study's design, data collection techniques, analysis techniques, and ethical procedures followed. Chapter 4 is the core of the thesis, presenting results and discussion of findings in four major sections, each representing an objective.

The first section presents the effect of instructional strategies on learners' attitudes toward transformation geometry. The second section investigates how instructional strategies impact learners' progression through the Van Hiele levels. The third section explores the predictive and mediating role of learners' attitudes in their progression through Van Hiele levels. The fourth chapter shifts focus to learner

challenges, support needs, and the perceived effectiveness of the two instructional strategies, providing contextualized evidence on how each strategy shapes learning and the learners' experiences.

Finally, Chapter 5 brings the thesis to a close by synthesizing the results, summarizing key findings and offering recommendations. The thesis concludes with references and appendices, which include research instruments, ethical clearance and approval documents, and other supporting materials.

CHAPTER TWO

LITERATURE REVIEW

In this chapter, the theoretical foundations, starting with a theoretical review, are presented; focusing on the existing literature, geometry education, and students' challenges in mastering geometric concepts. The examination encompasses the Van Hiele phase-based instructional strategy, elucidating its theoretical framework and how it addresses the cognitive development stages in geometric learning. Additionally, attention is directed towards the technology-enhanced instructional strategy, exploring how technology integration impacts geometry education outcomes.

A significant aspect of the chapter involves an in-depth analysis of learners' attitudes towards geometry learning and conceptual and procedural understanding. This chapter culminates in a discussion of the existing research gap, providing a concise summary of the reviewed literature, synthesizing key insights, and establishing a foundation for the subsequent chapters.

2.1 Theoretical Review

The theoretical foundation guiding this study is a synthesis of Constructivism and the Van Hiele theory. The two theories are integrated to offer a comprehensive lens for the study. Constructivism highlights the active nature of learning, emphasizing engagement, while the Van Hiele Theory adds a cognitive developmental perspective specific to geometric reasoning. Together, they guide the investigation of instructional strategies, their influence on geometry attitudes, and their effects on learners' progression through various Van Hieles levels.

2.1.1 Constructivist Learning Theory

Constructivism, as a theory of learning, has two primary forms: Cognitive Constructivism and Social Constructivism. Cognitive Constructivism, rooted in the work of Jean Piaget, emphasizes that learning is an individual, internal practice where knowledge is actively built based on prior experiences and cognitive development (Allen, 2022).

According to Piaget (1980), learners progress through distinct developmental stages, and understanding is built through the processes of assimilation (integrating new information into existing knowledge) and accommodation (modifying existing knowledge to incorporate new experiences). This theory suggests that learners actively engage with content, explore concepts independently, and develop deeper understanding through self-discovery.

In contrast, Social Constructivism, as proposed by Lev Vygotsky, emphasizes the role of social interaction, cultural context, and language in the learning process. It argues that knowledge is constructed through collaboration, discussion, and scaffolding, where more knowledgeable individuals (teachers or peers) provide guidance within the Zone of Proximal Development (ZPD) to help learners acquire skills beyond their independent abilities (Allen, 2022).

This study is grounded in both Piaget's and Vygotsky's constructivist theories, each guiding different but complementary aspects of the learning process. Piaget's cognitive constructivism provides the foundational conceptual basis for the Van Hiele Phased Instruction (VHPI), explaining the developmental stages through which learners' progress in geometric reasoning. The Van Hiele model aligns with Piaget's view of hierarchical cognitive development, which occurs in stages.

Conversely, Vygotsky's social constructivism informs the design and delivery of instructional phases, emphasizing the benefit of social interaction, scaffolding, and mediation in the process of learning. Through guided instruction, collaborative activities, and the use of mediational tools such as technology, learners' attitudes, affective, behavioral, and cognitive, are shaped and transformed. This

social constructivist perspective highlights how instructional strategies not only facilitate cognitive progression but also actively influence learners' motivation and disposition toward geometry. Together, these theories offer a comprehensive framework for comprehending both the nature of cognitive development in geometry (Piaget/VHPI) and the process by which instructional support and social factors drive learners' advancement and attitude change (Vygotsky).

In contrast to traditional classrooms, a constructivist classroom emphasizes engaging and transferable learning experiences, where learners actively participate in constructing knowledge. The teacher assumes the role of a facilitator. At the same time, learners often work in collaborative groups, reflecting a dynamic view of knowledge and valuing the process of learning as much as the product (Bada & Olusegun, 2015).

These features are strongly grounded in social constructivist theory, particularly Vygotsky's view that learning is socially mediated and culturally situated. While cognitive constructivism (as advanced by Piaget) also informs educational theory by highlighting individual developmental processes, social constructivism provides a more comprehensive lens for understanding classroom practices that emphasize interaction, scaffolding, and guided discovery (Alanazi, 2016).

2.1.2 Van Hiele's Theory of Geometric Reasoning

Pierre van Hiele and Dina van Hiele Geldof formulated the Van Hiele theory of geometric reasoning, driven by a desire to unravel learners' cognitive journey while mastering geometric concepts. The motivation for this theory arose from observing the challenges their students faced when dealing with geometry-related problems. This compelling motivation encouraged the theory's inception, crystallizing during their 1957 thesis at the University of Utrecht (Crowley, 1987; Vojkuvkova, 2012; Van Hiele, 1957).

Beyond merely shedding light on learners' difficulties in geometry lessons, the theory ventures into practical solutions, introducing an instructional approach to overcome these challenges and offering a valuable rubric for strategically placing learners at different cognitive levels (Vojkuvkova, 2012).

The theory defines five sequential and hierarchical levels: Visualization, Analysis, Abstraction, Deduction, and Rigor (Moru et al., 2021). Notably, the theory highlights that the progression between these levels is more intricately tied to educational experiences than the learner's age or maturity.

According to the Van Hiele theory, learners navigate through these cognitive stages via organized teaching or instruction, a process neatly divided into five learning phases: Information, Guided Orientation, Explication, Free Orientation, and Integration. Mastery of a particular topic mandates learners to cycle through these phases iteratively for comprehensive understanding (Wulandari et al., 2021; Machisi & Feza, 2021).

In addition to its systematic framework, the Van Hiele theory emphasizes the alignment between teachers and learners in terms of cognitive levels. It posits that effective progress hinges on this alignment, highlighting that misalignment can impede learners' advancement, resulting in rote memorization without genuine mastery. Acknowledging potential variations in geometric understanding across content strands, the theory asserts that achieving a specific thought level in one strand lays a foundation for easier attainment in another. This perspective adds depth to the theory's applicability and emphasizes its potential impact on diverse aspects of geometric education.

Although the Van Hiele model is widely applied in geometry education, it has been criticized for its lack of clear diagnostic tools to determine when a learner transitions from one level to another (Usiskin, 1982). Learners may also display characteristics of multiple levels simultaneously, challenging the model's strict hierarchical assumption (Alex & Mammen, 2016).

Despite these limitations, the model remains a valuable framework for structuring instruction when supported by diagnostic tasks and formative assessments, as applied in this study through the Van Hiele Geometry Achievement Test (VGAT) and classroom observations.

2.1.3 Synthesizing Constructivism and the Van Hiele Theory in Geometry Education

This study is grounded in a combined theoretical framework of Constructivist Learning Theory and the Van Hiele Theory of Geometric Reasoning. The cognitive dimension, informed by Piaget's theory, explains the developmental stages of geometric thinking, which are operationalized through learners' progression across the Van Hiele levels (Allen, 2022).

Complementing this, the social dimension, based on Vygotsky's social constructivism, guides the design and implementation of instructional strategies, specifically the Technology-Enhanced Van Hiele Phased Instruction and the Conventional Van Hiele Phased Instruction. Vygotsky's framework also provides a basis for understanding how learners' attitudes toward geometry are shaped through social interaction and mediated learning experiences (Aldiabat & Yew, 2024; Uygun & Guner, 2021)

In this model, learners' attitudes are influenced by both the instructional strategies and the learners' experiences with those strategies. These attitudes then mediate learners' cognitive progression through the Van Hiele levels.

Thus, the conceptual framework positions instructional strategies as the independent variable, learners' experiences and attitudes as mediating variables, and progression through Van Hiele levels as the dependent variable. Figure 5 illustrates the synthesis of the two theories and how they support geometry instruction.

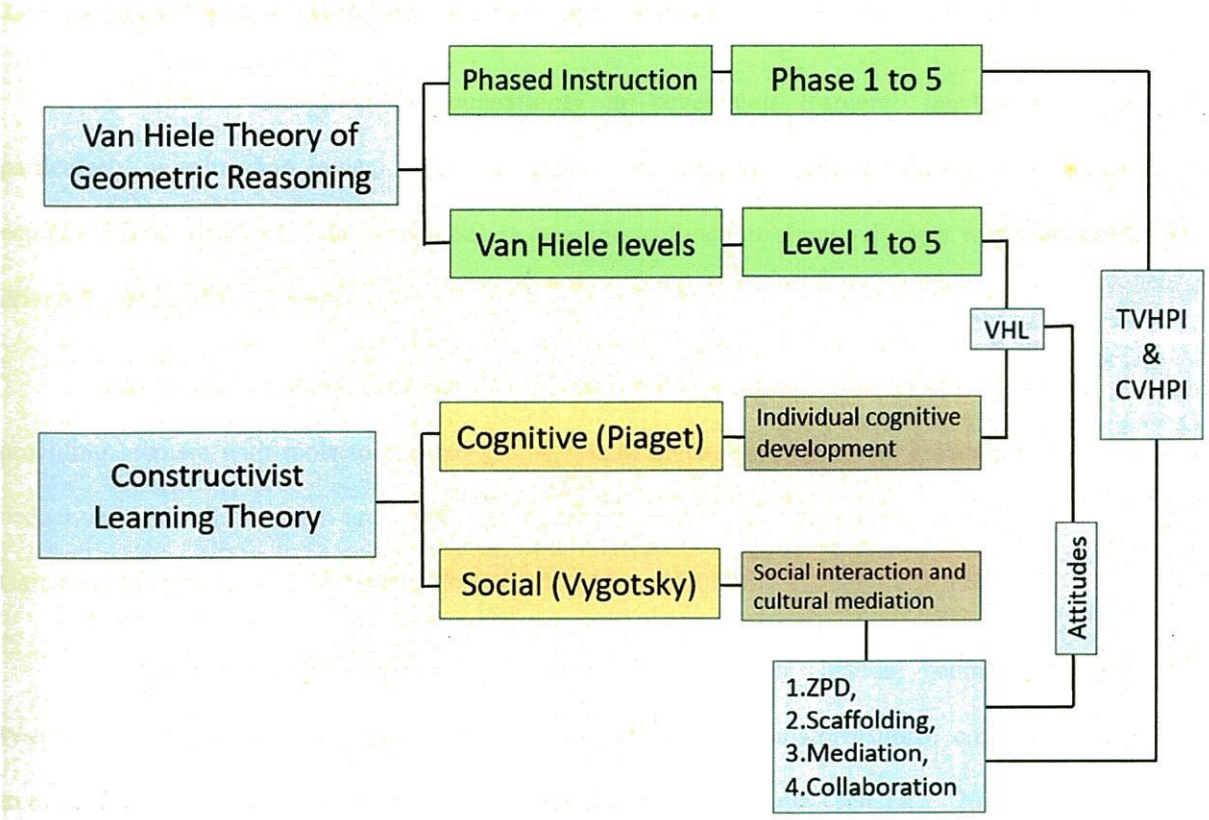


Figure 5: Showing the synthesis of the constructivist and Van Hiele Theories to inform instruction.

Existing research often focuses on the effectiveness of either constructivist approaches or the Van Hiele framework in isolation, without exploring how the two interact to enhance learning. This study addresses this theoretical gap by explicitly linking Cognitive Constructivism and the Van Hiele model to analyze how instructional interventions shape both the process of knowledge construction and the structured progression of geometric reasoning.

By synthesizing these two perspectives, the study provides a more coherent theoretical foundation for understanding the mechanisms through which instructional strategies support cognitive and geometric development in transformation geometry.

2.2 Mathematics Education and Instructional Strategy

Mathematics education is foundational in developing learners' intellectual capabilities, particularly in enhancing logical reasoning, problem-solving, and critical thinking. It is important in equipping individuals with the competencies required to function effectively in a world dominated and driven by data and technology.

Among the key domains of mathematics, geometry occupies a distinctive and integral position, providing learners with tools to explore spatial relationships, develop visual reasoning, and engage in deductive logic, all of which are crucial for success in various scientific and practical contexts (Liñán-García et al., 2021; Sinclair & Bruce, 2015; Smith & Jones, 2020).

In the mathematics curriculum for secondary school in Uganda, geometry is introduced systematically and developed across various topics, including transformations, constructions, angles, trigonometry, coordinate geometry, and three-dimensional shapes (NCDC, 2019a, 2019b). The curriculum aligns with the Competency-Based Education framework, emphasizing learner-centered approaches, real-life application, knowledge and skills development, positive values, and attitudes.

Despite these curricular intentions, evidence from national examination reports and recent studies (UNEB, 2014, 2016, 2017a, 2017b, 2018, 2019, 2020, 2021a, 2021b, 2023) indicates that learners continue to face considerable challenges in mastering geometry. These challenges are manifested in poor examination performance, shallow conceptual understanding, and difficulties in integrating geometric reasoning with other mathematical domains such as algebra (Ndungo et al., 2024).

The persistent underperformance in geometry can be partly connected to the nature of instruction. Traditional teaching approaches that emphasize memorization of procedures often fail to support the conceptual understanding for learners to internalize geometric principles. In contrast, contemporary pedagogical models such as the Van Hiele Theory of Geometric Reasoning provide a structured framework for teaching and learning geometry (Abdullah & Zakaria, 2013; Larbi, 2021; Tahani, 2016).

The theory proposes that learners progress through distinct, hierarchical levels of understanding, ranging from basic visualization to advanced deductive reasoning. Each level builds upon the previous one, and progression requires targeted instructional support aligned with learners' cognitive readiness.

Studies have shown that instructional approaches grounded in Van Hiele theory, such as Van Hiele Phased Instruction (VHPI), can effectively guide learners through these levels. When enhanced with technological tools like GeoGebra, this approach, referred to as TVHPI, offers interactive and dynamic visualizations that promote deeper engagement and a better understanding of geometric concepts (Mthethwa et al., 2020a; Narh-kert & Sabtiwu, 2022; Zengin, 2017).

These strategies also support the management of cognitive load by allowing learners to explore, manipulate, and visualize abstract geometric relationships in meaningful ways. However, their application in Ugandan classrooms remains limited, and comparative evidence on their effectiveness is still emerging. Given the central role of geometry in mathematics education and its contribution to developing learners' overall mathematical reasoning, it becomes crucial to examine instructional strategies that are both theoretically robust and adaptable to specific contexts.

This study sought to address that need by comparing the effects of CVHPI and TVHPI on learners' attitudes and their progression in transformation geometry, while also investigating learners' lived experiences, challenges, and perceptions of these instructional approaches.

2.3 Instructional Strategies and Learners' Progression through Van Hiele's Levels

2.3.1 Van Hiele's Phased Instruction and Geometry Education

This section explores how Van Hiele's phases are used in teaching geometry. These offer a structured approach to understanding the progression of students' geometric thinking. These levels offer educators

valuable insights into students' cognitive development, informing the design of effective instructional strategies.

Studies have been conducted to establish the effect of van Hiele's phased instruction on learners' achievement in geometry. For example, a study by Abdullah and Zakaria (2013) found that students' level of geometric thinking improves when taught using the van Hiele phased instruction compared to traditional instructional strategies. Their study found that only one student from the experiment failed to achieve level 3. Additionally, at the initial stage of their experiment, it was found that all the students had achieved the first van Hiele level of geometric thinking. They concluded that activities related to geometry can be implemented and organized most effectively using Van Hiele's phased instruction. Their study utilized Geometer's Sketchpad software as the implementation medium, with the topic of interest being transformation geometry. Given the diverse range of geometry concepts and the existence of other geometry dynamic software that can serve as implementation media, in addition to Geometer's Sketchpad, an opening for the current study exists.

To mitigate the challenges learners face in learning geometry, van Hiele's phased instruction has been recommended for designing and implementing geometry teaching programs and lessons. Teachers who have used this strategy have found that phased instruction is an excellent pedagogical approach for planning and presenting lessons. In addition, countries that have adopted van Hiele's theory of geometric thought, such as Taiwan, Ghana, Malaysia, and South Africa, have registered high-level performance in mathematics (MdYunus et al., 2019). For example, a study in Ghana conducted to find the effect of Van Hiele phased approach on students' achievement in circle geometry at a senior high school indicates that students moved from the pre-recognition level before the intervention to level 2 after the intervention. It was thus concluded that the strategy facilitated learning, and teachers were encouraged to understand the geometric reasoning levels of their learners before instruction began (Yalley et al., 2021).

With these experiences, the current study adopts van Hiele's phased strategy and investigates its influence on learners' geometry attitudes and progression through Van Hiele levels in the Ugandan context.

2.3.2 Technology-enhanced Instructional Strategy and Geometry Education

2.3.2.1 Technology-Integration in Mathematics Education

In recent years, the incorporation of technology into education has grown rapidly, reflecting curriculum reforms that promote active, participatory learning through the use of interactive digital tools, including tablets, smartphones, and specialized software (Diaz-Nunja et al., 2018). In mathematics classrooms, access to ICT resources enriches learning by enhancing engagement, supporting exploratory problem-solving, and encouraging discovery-oriented approaches (Mosese & Ogbonnaya, 2021). This shift is particularly evident in the teaching of geometry, where both Uganda's new lower secondary curriculum and emerging empirical research highlight the importance of integrating technological applications into instructional practice (NCDC, 2019b; Ndungo, Nazziwa, et al., 2025).

Given the wide-ranging benefits of incorporating technology into mathematics education, such as expanding learning opportunities, boosting student engagement, and fostering discovery-oriented learning, a variety of specialized software tools have been created specifically to support these goals. These include GeoGebra, MATLAB, Mathematica, and Geometers' Sketchpad (Praveen & Leong, 2013). In addition, research has examined the application and impact of these software tools in mathematics instruction, evaluating how effectively they support teaching and enhance learning outcomes (Bedada & Machaba, 2022).

Among these software programs, GeoGebra is one of the most studied software programs. However, these studies are not exhaustive, and the effectiveness of GeoGebra remains questioned in some

other contexts (Mosese & Ogbonnaya, 2021). For example, it is not widely used in teaching and learning mathematics in Uganda. The current study investigated the use of GeoGebra integrated with Van Hiele's phased instruction in the teaching and learning transformation geometry among senior three learners.

The use of technology in the teaching and learning of mathematics has been recommended as very important. It is, however, pointed out in several studies that the integration of technology in teaching and learning largely depends on the availability of the required software. Also, free technologies that are readily available on the Internet are more commonly used than those on the market. GeoGebra, as a free and accessible software, offers a practical solution for both teachers and learners to use directly in the classroom.

In Uganda, the revised lower secondary school mathematics curriculum underscores the importance of integrating such technological tools into instruction, aiming to make mathematics teaching more interactive, engaging, and aligned with modern educational practices (NCDC, 2020). GeoGebra is one of the most accessible tools teachers and learners can use to implement this curriculum to better their performance and understanding of the subject (Bekene, 2020). In this regard, the current study investigated the potential of GeoGebra in providing an opportunity to mathematics teachers for integrating technology in the teaching and learning of mathematics with confidence and empirical evidence that the approach yields positive learning outcomes.

2.3.2.1 Effectiveness of GeoGebra in Mathematics Instruction

A study investigating learners' understanding of circles using GeoGebra found that learners perform better when using GeoGebra and positively perceive using GeoGebra in learning circle geometry (Praveen & Leong, 2013). Additionally, using GeoGebra attracted, motivated, and encouraged learners to learn independently, unlike in the conventional method of teaching and learning geometry, where the chances of giving up were high (Mollakuqe et al., 2020). This finding aligns with Celen's (2020) observation that using GeoGebra software in teaching and learning makes mathematics more engaging and helps learners

develop mathematical generalizations. However, learners with limited computer knowledge find it challenging to use GeoGebra.

The aforementioned limitation can be mitigated by providing introductory lessons on using the software to learners. The current study investigated the metacognitive and self-regulated learning modes adopted by learners during instruction, guiding how to adapt different instructional strategies.

According to Mosese & Ogbonnaysa (2021), integrating GeoGebra into geometry instruction is a learner-centered approach that aligns closely with constructivist theory, which asserts that knowledge is actively built in the learner's mind through direct participation. In this framework, learning becomes an active process where students are fully engaged, exploring, manipulating, and reflecting, rather than passively receiving information. Wassie and Zergaw (2019) recommend using GeoGebra to supplement the conventional approach, rather than replacing it. This study integrates GeoGebra with the Van Hiele Phased instruction to provide a dynamic and engaging strategy of teaching geometry that leverages technology, thereby facilitating conceptual development among learners.

Similarly, a study by Mudaly and Fletcher (2019) on the effectiveness of GeoGebra in teaching linear functions using of GeoGebra supports learners in determining the properties of straight-line graphs, and learners showed a positive attitude toward the application. Additionally, a study by Yildiz and Baltaci (2016) found that using GeoGebra can facilitate the learning of cylindrical and spherical coordinates in a meaningful and lasting manner. To extend on these prior studies, the current study integrated GeoGebra with the van Hiele phased instruction to assess its effect on learners' level of geometric thinking and their attitudes toward geometry.

One of the essential student engagement strategies in teaching geometry, called " REACT" (relating, experiencing, applying, cooperating, and transferring), was used in tandem with GeoGebra by Yildiz and Baltaci (2016), and it was found to be an effective tool. Conversely, the current study used GeoGebra in tandem with Van Hiele's phased instruction to ascertain if integrating the two yields better geometry achievement and development of Van Hiele's levels of geometrical reasoning.

In other related studies, GeoGebra was found to be an effective tool for improving students' visualization. Its use in teaching and learning geometry helps learners to create correct visual images, acquire spatial abilities, and support students' mathematical communication, which are essential mathematical thinking abilities required in geometry (Vágová & Kmetová, 2019). Due to this, mathematics teachers are encouraged to use GeoGebra in teaching geometry (Kusumah et al., 2020). Although the use of GeoGebra requires the availability of computers in schools, it affects learners' learning. It has a positive effect on the teachers' beliefs regarding teaching and learning, even in rural areas where technology is limited (Mthethwa et al., 2020b).

Geometry skills, such as cognitive activities, reasoning, and visualization, were found to improve when using GeoGebra software in teaching mathematics (Diaz-Nunja et al., 2018). Relatedly, Pavethira and Leong (2017) found a significant difference in the scores between genders and between the post-test and pre-test, implying that using GeoGebra enhances students' performance in geometrical studies. This study suggests that teaching and learning with GeoGebra would help learners explore the concepts in more detail and build concrete knowledge in geometry. The current study evaluated the effect of combining GeoGebra with Van Hiele's phased instruction on learners' progression through Van Hiele levels and geometry attitudes at the secondary school level.

2.3.2.1 Need for Contextualized Instructional Innovation in Geometry Education

A study investigating the impact of GeoGebra on students' ability to connect different representations of trigonometric functions with their corresponding graphs reported a statistically significant difference in mean achievement between the experimental group and the control group. The findings indicate that incorporating GeoGebra into trigonometry lessons enhanced learners' performance compared to using traditional teaching methods, suggesting that the software supports deeper conceptual understanding and more effective learning. The study recommended that further research be done to determine the impact of

GeoGebra-based instruction on learners' progression with a large sample of learners and various topics in mathematics (Mosese & Ogbonnaya, 2021).

The current study extends the findings of this research to the use of GeoGebra integrated with the Van Hiele phase in teaching and learning transformation geometry, as most schools have not yet implemented instructional technologies, which could be due to a range of factors, such as the availability of the technology and teachers' ability to use it (Pavethira & Leong, 2017). This situation leaves a gap regarding whether these technologies could improve learners' achievement, progression through Van Hiele levels, and geometry attitudes.

The current study utilized a readily available tool to close this gap. In addition, the current problem of learners' low achievement levels in geometry at UCE could be addressed through the use of the GeoGebra tool in the teaching and learning of geometry since it has the potential to create opportunities for learners to engage and discover for understanding during lessons compared to traditional methods that often rely on static diagrams, rote procedures, and teacher-centered demonstrations, which may limit learners' conceptual development and active exploration.

2.4 Instructional Strategies and Learners' Attitudes toward Transformation Geometry.

2.4.1 Learners' Attitudes towards Geometry

Learners' attitudes refer to their beliefs, emotions, and dispositions toward a subject, shaping their motivation, engagement, and learning experiences (Bora & Ahmed, 2018). In mathematics education, attitudes are critical in determining how students approach, engage with, and persevere in learning geometry. Attitudes are generally classified into three key constructs: affective, behavioral, and cognitive attitudes. Affective attitudes relate to students' interest, confidence, and emotional responses toward the subject of geometry. Behavioral attitudes reflect their participation, effort, and engagement in learning

activities. Cognitive attitudes encompass their perceptions of the relevance and applicability of geometry in real-world contexts (Smith & Jones, 2020).

Studies suggest that learners' attitudes toward geometry vary widely, with some students recognizing its importance while others struggle with its abstract nature (Naidoo & Kapofu, 2020). Research on gender differences presents conflicting findings; some studies report no significant differences (Sunzuma et al., 2013), while others indicate that female students exhibit more positive attitudes (Arslan et al., 2012). These inconsistencies highlight the need for further research into the factors influencing students' attitudes in geometry.

Technology, particularly GeoGebra, has been found to enhance students' affective, behavioral, and cognitive attitudes toward mathematics by improving visualization, interaction, and conceptual understanding (Uwurukundo et al., 2021). However, most studies have focused on a single instructional approach, leaving gaps in understanding how different methods compare in shaping students' attitudes. Research also suggests that students who perceive geometry as relevant to real-world applications develop greater cognitive engagement and motivation (Smith & Jones, 2020). However, gradual attitudinal changes require instructional strategies that actively engage students across all three constructs (Albarracin & Shavitt, 2018).

While the Van Hiele Phased Instruction model is widely recognized for enhancing students' geometric reasoning, its impact on attitudes remains underexplored (Larbi, 2021; Naufal et al., 2021; Ndungo, 2024; Yalley et al., 2021; Zhou et al., 2022). Similarly, despite the potential of Technology-Enhanced Van Hiele Phased Instruction to improve engagement and motivation, research on its implementation in Uganda remains limited. This study addresses these gaps by comparing TVHPI and CVHPI, analyzing their influence on learners' affective, behavioral, and cognitive attitudes toward transformation geometry.

2.4.2 Attitudinal Changes among Learners

Measuring changes in attitude over time presents challenges, particularly when attitudes are viewed as fixed and immutable, implying a stable and enduring perspective. Conversely, measuring change becomes more feasible when attitudes are seen as dynamic and influenced by immediate circumstances, such as mood (Albarracín & Handley, 2011). This distinction is crucial for understanding how instructional strategies impact learners' attitudes.

The current study examines how learners' attitudes change when they are exposed to different instructional strategies. Despite the potential for change, most research indicates that such shifts are modest, typically around one-third of a standard deviation (Albarracin & Shavitt, 2018). This suggests that while instructional strategies can influence attitudes, their effects may be incremental and vary depending on the intervention's context and nature (van Bavel et al., 2012). The current study aims to explore these shifts in the context of varying instructional strategies, with the goal of better understanding their impact on learners' attitudes.

To effectively measure and analyze changes in attitude, researchers can utilize several methods, including total scores, item-level changes, or a qualified shift algorithm. The shift algorithm is preferred for its detailed analysis, as it captures various influencing factors and offers a comprehensive view of attitude changes (Trachtenberg et al., 2007). For this study, applying the algorithm involved collecting pre- and post-test data, developing and implementing the algorithm, analyzing the shifts, and interpreting the results to evaluate the impact of instructional strategies on learners' attitudes. The qualified change algorithm quantified significant shifts in learners' attitudes by setting thresholds for meaningful change, categorizing responses, and calculating composite scores across different constructs (Affective, Behavioral, and Cognitive).

Lastly, attitudes are typically not uniform; they differ in strength and stability. Early-formed symbolic attitudes often stem from emotional responses, whereas later-formed symbolic attitudes are based on more deliberate reasoning and thought. Negative attitudes generally resist change more than

positive ones (Petty, 2012). Educational interventions and learning experiences play a crucial role in altering attitudes. This understanding is crucial for the study, as it highlights that changing attitudes through instructional strategies involves addressing both the emotional and cognitive components of learners' beliefs. Therefore, Effective instructional methods must consider learners' developmental stage and the inherent difficulty in shifting negative attitudes.

The current study addresses key gaps in existing literature on Van Hiele Phased Instruction (VHPI) and attitudes toward geometry. While VHPI enhances geometric understanding, its impact on attitudes remains underexplored. The long-term link between attitudes and performance is also insufficiently studied. Additionally, the integration of technology in Ugandan geometry education is limited. Previous research often focuses on gender differences rather than broader influencing factors. This study examines how various instructional strategies shape learners' attitudes toward transformation geometry in the Ugandan context.

2.5 Learners' Challenges and Understanding in Learning Geometry

2.5.1 Learners' Challenges in Learning Geometry

According to Ngirishi and Bansilal (2019), many learners struggle to grasp foundational ideas in geometry, such as interpreting geometric properties and identifying various shapes. Consequently, they often remain at lower stages of geometric reasoning, which restricts their ability to comprehend more sophisticated concepts and advance to higher levels of geometric reasoning.

The findings indicate that many students rely on less developed forms of geometric reasoning, limiting their ability to grasp more advanced concepts. In addition, teachers often depend on traditional instructional approaches that may not address the varied learning needs of their students. A considerable number of learners face challenges in applying formulas and theorems, interpreting arguments, and understanding geometric terminology, which can make the subject seem difficult and unappealing them

(Kivkovich & Chis, 2016). Such outdated instructional practices risk reducing student engagement and impeding the development of essential foundational geometry skills.

Luneta, (2015) notes that many students face difficulties when working with mathematical instruments such as compasses and protractors, which can result in unanswered questions and create further hurdles in solving problems. To address these challenges, Moru et al. (2021) advocate for a deliberate focus on strengthening learners' conceptual understanding alongside the adoption of effective teaching strategies. By improving the way geometry is taught, students can develop a deeper grasp of the subject and enhance their overall mathematical competence.

2.5.2 Strategies for addressing Learners' challenges in Learning Geometry

Researchers have explored a range of instructional approaches to strengthen students' geometric reasoning and address existing challenges. Among these, the Van Hiele Phased Instruction model is recognized for its structured methodology in teaching geometry. It systematically guides learners from basic recognition of shapes to more advanced reasoning about geometric properties and transformations, following five sequential levels of understanding. While the Conventional Van Hiele Phased Instruction has demonstrated effectiveness in fostering comprehension, there is a growing movement toward integrating technology into this model to further enhance learning outcomes (Machisi & Feza, 2021; Zalman, 1982).

Technology-enhanced Van Hiele Phased Instruction, incorporating tools such as GeoGebra, offers learners the chance to engage actively and visually with geometric ideas. Through GeoGebra's dynamic visualization capabilities, students can manipulate shapes, witness transformations as they occur, and investigate geometric relationships in a more concrete manner. This interactive approach can help bridge the gap between students' current understanding and the abstract nature of geometric concepts,

making transformation geometry more accessible and engaging (Adelabu et al., 2022; Iannone & Miller, 2019; Mthethwa et al., 2020; Vágová & Kmetová, 2019).

Research has consistently shown that GeoGebra can enhance both geometric reasoning and learner engagement. For instance, Abdullah and Zakaria (2013) reported that students who learned using dynamic software such as GeoGebra demonstrated significantly greater gains in geometric understanding compared to those taught through traditional approaches. Additional studies further reveal that GeoGebra supports visualization, spatial reasoning, and problem-solving skills, thereby making mathematics more interactive and enjoyable (Mollakuqe et al., 2020; Celen, 2020). Its effectiveness extends across a range of topics, including circles, linear functions, 3D geometry, and trigonometry, enabling learners to investigate mathematical concepts in greater depth and with improved comprehension (Mudaly & Fletcher, 2019; Uwurukundo et al., 2021; Yildiz & Baltaci, 2016).

In recent decades, the teaching of geometry has undergone significant shifts, with a growing focus on building conceptual understanding rather than relying solely on memorizing procedures (Clements & Sarama, 2021). While traditional approaches often centered on formal deduction, this method has proven less effective for younger learners or those without prior exposure to geometric concepts (Van de Walle et al., 2016). To prepare students for grasping more advanced topics such as transformation geometry, it is essential to cultivate spatial reasoning and visualization skills from an early age (Sinclair & Bruce, 2015).

Despite these developments, research continues to highlight persistent difficulties in effectively teaching transformation geometry, especially in settings where students find abstract ideas, such as rotations, reflections, and dilations, challenging to grasp (Sunzuma & Maharaj, 2019). Zeinul and Silmi (2022) observed that two major hurdles learners face are applying geometric theorems and solving complex geometry problems. Their findings show that activity-based instructional approaches can help address these issues, enhancing both students' understanding and their engagement with the subject.

Building on this foundation, the present study investigates how instructional approaches such as CVHPI and TVHPI contribute to improving learners' understanding of transformation geometry. While Zeinul & Silmi (2022) emphasized the use of active learning strategies, this research extends the discussion by assessing the added value of integrating technology, specifically GeoGebra, into the instructional process. By exploring technology-enhanced learning within the Ugandan context, the study addresses an important knowledge gap and provides insights into how digital tools can help overcome persistent challenges in teaching geometry.

2.5.3 Learners' Conceptual and Procedural Understanding of Geometry

The balance between conceptual and procedural understanding has historically posed a challenge in many educational contexts. Procedural knowledge has been the priority of many mathematics teachers due to systemic constraints and curriculum demands. This procedural dominance has persisted despite teachers' intentions to teach for understanding, illustrating the gap between theory and classroom practice.

However, the new curriculum being implemented in Ugandan lower secondary schools emphasizes that both conceptual and procedural understanding are balanced to produce a holistic learner who is a critical thinker, problem solver, and innovator (NCDC, 2020). According to Chappell and Killpatrick (2003), conceptual learning could be enhanced without sacrificing procedural rigor. The current study examined how the two instructional strategies shaped learners' conceptual understanding, measured by the Van Hiele levels of geometric reasoning.

The study by Bahr and Bossé (2008) encourages the integration of both procedural and conceptual knowledge among educators to facilitate teaching and learning, aligning with Star (2020), who also highlighted the interdependence of the two types of knowledge in the teaching and learning of mathematics.

Research has been conducted to establish the best teaching approach that can balance the two types of knowledge. For example, Lestari and Surya (2017) investigated Realistic Mathematics Education (RME), Ploger and Hecht (2009) investigated the Chartworld software, and Malatjie and Machaba (2019) explored the use of concept mapping to enhance understanding of transformation geometry; all highlighting that these approaches significantly facilitate the balance between the two kinds of knowledge. Following these, Asfar and Asfar (2020) introduced Case-Based Games Learning (CBGL) with Quizizz, revealing higher engagement and improved understanding compared to traditional case-based methods. This highlights a shift toward interactive and technology-based approaches to address the challenges of conceptual understanding.

Despite their value and contributions to the field of mathematics education, these studies are limited in their generalizability due to their focus on a single topic of mathematics and their contextualization outside Uganda. This highlights the need for further research that integrates visual tools and other instructional strategies that may supplement and close the existing gaps in the Ugandan context.

Recent research emphasizes technology-based strategies. For example, Kanive et al. (2014) highlighted the benefits of computer-based practice in improving retention and computational fluency among struggling learners. The benefit, however small, facilitates deep learning as long as it facilitates representation and engagement (Richland et al., 2012).

More recent studies, such as Mendezabal and Tindowen (2018), demonstrated the potential of Microsoft Mathematics to improve attitudes and skills in Differential Calculus. While these tools foster engagement, their long-term impact on conceptual understanding remains underexplored, especially in comparative contexts.

The relationship between conceptual and procedural knowledge was studied by Star (2000), while Kusumaningsih et al. (2019) examined gender differences in conceptual understanding. Additionally, Salim and Gilar (2020) demonstrated that balanced competencies significantly improved problem-solving abilities in fractions. Similarly, Chappell and Killpatrick (2003) found that concept-based instruction

enhanced understanding without compromising procedural skills. Both studies emphasize the need for tailored interventions for individual cognitive styles and learning needs. Meanwhile, the broader literature reveals a gap in comparative studies explicitly evaluating instructional strategies for balancing these competencies. Most studies either focus on one form of knowledge or examine their relationship in isolation, leaving unanswered questions about how best to integrate them.

This study addresses these gaps by investigating how Technology-Enhanced Van Hiele Phased Instruction compares to Conventional Van Hiele Phased Instruction in shaping learners' conceptual and procedural understanding.

2.6 The Research Gap

The literature review identifies four key gaps: theoretical, conceptual, methodological, and contextual gaps in the application of instructional strategies in transformation geometry.

The limited integration of technology in constructivist and the Van Hiele Theory defines the theoretical gap that needs attention. Although Van Hiele's theory of geometric thinking is key for structured instruction, and the constructivist learning theory provides for active learning and learner-centered pedagogies, neither of them guide the digital tools that are appropriate for instruction, yet technology integration in teaching and learning is paramount and a trend in the education landscape worldwide.

Similarly, the two theories did not take into account the role of other facilitators of learning, such as learners' affective, behavioral, and cognitive characteristics. The current study closes this theoretical gap by integrating technology and exploring how the teaching supports conceptual development within constructivist and Van Hiele's theoretical frameworks and how learners' attitudes mediate their progression levels.

Additionally, the Van Hiele model is limited to general geometry; its application to specific and yet essential topics like transformation geometry remains underexplored. Moreover, attitude shifts in cognitive development are not thoroughly investigated, creating a crucial gap addressed in this study.

Most of the studies on Van Hiele levels relied on quantitative pre-posttest methods, and little attention was given to mixed methods that incorporate qualitative data to capture students' learning experiences and the context under which the instructions are implemented. The current study aims to close this methodological gap by employing mixed methods, involving a quasi-experiment, pre-test and post-test data, and post-intervention interviews, to understand learners' experiences, challenges, and the support required during the learning process.

Lastly, in Uganda, studies on geometry instruction rarely apply the Van Hiele model beyond traditional teacher-centered approaches. Technology-Enhanced Van Hiele Phased Instruction has received almost no local investigation, and the mediating or predictive role of learners' attitudes in progression through Van Hiele levels remains unexplored. This lack of context-specific evidence limits the effective implementation of the Competency-Based Curriculum and ICT-supported pedagogy in secondary schools.

In response, this study developed hypotheses and questions aligned with four key objectives: to assess shifts in learners' attitudes, evaluate their cognitive progression, explore the predictive and mediating role of attitudes, and understand learners' experiences with the instructional models. These focus areas were selected to fill gaps in both theory and practice, offering a multidimensional investigation into how instructional strategies can be optimized to improve the teaching and learning of transformation geometry in Ugandan secondary schools.

CHAPTER THREE

METHODOLOGY

This chapter outlines the research procedures followed in the study and the rationale for each methodological decision. It begins by discussing the study's research paradigm and philosophical foundations. It then details the research design, population, sample size, sampling plan, and the tools used for data collection. The data analysis procedures are also detailed according to the objectives, and finally, the ethical principles adhered to in the study are described.

3.1 Research Paradigm and Philosophical Assumptions

This study is anchored in the Pragmatic Paradigm, which advocates for the integration of quantitative and qualitative approaches to generate a comprehensive understanding of complex educational phenomena. Pragmatism emphasizes the utility of methods based on their capacity to address research questions effectively rather than adhering to rigid philosophical positions (Creswell & Creswell, 2018). Four key philosophical assumptions underlie this paradigm: ontological, epistemological, axiological, and methodological assumptions.

3.1.1 Ontological Assumption (Nature of Reality)

Ontology concerns the nature of reality and what constitutes existence (Saunders et al., 2019). This study assumes that reality is both singular and multiple. On one hand, constructs such as learners' attitudes toward geometry and their Van Hiele levels of geometric reasoning are considered measurable realities that can be observed objectively through standardized instruments. On the other hand, learners' interpretations, experiences, and interactions with instructional strategies are socially constructed and

vary across individuals. Thus, this research acknowledges the coexistence of objective and subjective realities, which aligns with the pragmatic stance of embracing multiple perspectives.

3.1.2 Epistemological Assumption (Nature of Knowledge)

Epistemology relates to the nature, sources, and validation of knowledge (Saunders et al., 2019). This study assumes that knowledge is generated through empirical observation and human interpretation. The quantitative phase produced measurable evidence through attitude scales and Van Hiele diagnostic tests, whereas the qualitative phase elicited deeper insights through interviews and reflective narratives. By integrating both forms of evidence, the study adheres to the pragmatic notion that no single approach provides a complete understanding but that knowledge is best constructed through the convergence of different forms of inquiry.

3.1.3 Axiological Assumption (Role of Values)

Axiology addresses the role of values and ethics in research (Saunders et al., 2019). This study recognizes that research is value-laden. The researcher's beliefs influenced the framing of research questions and the interpretation of qualitative findings. To minimize bias and ensure credibility, reflexivity was maintained through a research journal, and trustworthiness was enhanced through triangulation, member checking, and adherence to ethical guidelines. Furthermore, the study prioritized respect for participants' voices and confidentiality, reflecting a value-driven approach consistent with qualitative inquiry.

3.1.4 Methodological Assumption (Approach to Inquiry)

Methodology concerns the strategies employed to acquire knowledge. Guided by pragmatism, this study adopted a Mixed-Methods Sequential Explanatory approach to inquiry, which first collected and analyzed quantitative data to establish patterns in learners' attitudes and Van Hiele progression. This was followed

by a qualitative phase to explain and contextualize these patterns through participants' experiences. This methodological choice reflects the belief that integrating numeric trends with rich narrative accounts provides a holistic understanding of instructional effectiveness, in line with the pragmatic commitment to practical problem-solving.

3.2 Research Design

A quasi-experimental research design was employed to evaluate the effectiveness of the two instructional strategies on learners' attitude shift and progression through Van Hiele levels within the competency-based Curriculum framework. In this experiment, the research subjects are not randomly assigned to either group (Park et al., 2020). This made it an appropriate design for this study since the subject of the study consisted of learners who could not be randomly assigned to groups anymore, since they were already organized in class streams. The design included the following components:

Experiment 1 (conventional Van Hiele Phased Instruction, CVHPI) group: Learners in this group received VHPI, which, while serving as a structured intervention in this study, is also well aligned with the Competency-Based Curriculum implemented in the study schools and holds potential to strengthen its classroom application through its emphasis on conceptual progression, active engagement, and learner-centered instruction. Experiment 2 (Technology-Enhanced Van Hiele Phased Instruction, TVHPI) consisted of learners receiving TVHPI that integrates GeoGebra into the conventional variant to enhance outcomes.

Although quasi-experiments are classified as quantitative designs, Creswell and Creswell (2018) stress that primary mixed-method designs, such as convergent, exploratory, and explanatory designs, can augment an experiment by collecting qualitative data to support the quantitative findings. This study adopted a sequential explanatory mixed-methods design, where quantitative data collection preceded

qualitative data collection, with the latter serving to explain and elaborate on the former. The arrangement of the quasi-experiment per school is illustrated in Table 1.

Table 1: Arrangement of quasi-experiments for the study in a typical school

Group	Pre-test	Treatment	Post Test	Follow-up
CVHPI Class	VGAT1, TGAS1	Treatment (C)	VGAT2, TGAS2	Interviews
TVPHI Class	VGAT1, TGAS1	Treatment (T)	VGAT2, TGAS2	Interviews

Table 1 summarizes the quasi-experimental design of this study, with participants assigned to either treatment C (Conventional Van Hiele Phased Instruction) or treatment T (Technology-Enhanced Van Hiele Phased Instruction). Both groups completed pre-tests measuring geometry achievement (VGAT1) and attitudes toward transformation geometry (TGAS1), followed by their respective treatments. Post-tests (VGAT2 and TGAS2) and follow-up interviews were then conducted to assess changes and gather qualitative insights.

Although this study adopted a quasi-experimental design without full randomization, internal validity threats were mitigated through matched sampling of schools and classes, baseline pretesting of learners' attitudes and Van Hiele levels, and standardized instructional procedures across groups. These measures minimized selection bias and ensured comparability between the experimental and control groups.

3.3 Population, Sampling Procedures, and Sample Size

3.3.1 Population

Quasi-experiments are conducted in a population of closely identical individuals regarding the dependent and cofounding variables due to the limitation of randomizing participants (Scher et al., 2015). The

study's population was the senior three (S.3)[2] learners from secondary schools in a rural and urban district in the Rwenzori sub-region, western Uganda

Rural schools were selected from Kasese district, which had 98 secondary schools in this category, with an average of approximately 80 students in S.3 per school. On the other hand, the urban schools were selected from Fort-Portal, which had 19 schools in this category with an average of approximately 150 students in S.3 per school. This yielded a total population of 10,690 S.3 students across the two districts (April 2024). The information is summarized in Table 2.

[2] In Uganda, Senior Three (S.3) is the third year of lower secondary school, typically comprising learners aged 14 –17 years, who are preparing for the Uganda Certificate of Education (UCE) examinations.

Table 2: Approximate Population of S.3 students in the Study areas

Districts	No. of Sec. schs.	Average no. of students in S.3	Total students in S.3 for all schools
Kasese(Rural)	98	80	7,840
Fort Portal Tourism City(Urban)	19	150	2,850
Total	**107**		**10,690**

Source: District Education Department Records, Kasese and Fort Portal.

The choice of Senior Three (S.3) learners for this study is strongly justified by several key factors. Firstly, S.3 students have already covered foundational geometry material and were at a stage where they possess a certain level of geometric reasoning, equipping them with the necessary background to engage meaningfully with more advanced geometric concepts. Moreover, the geometry concepts at the S.3 level are notably more complex and introduce new material, particularly relevant to elements assessed in national exams such as UNEB. Additionally, the maturity level of S.3 learners is conducive to conducting experiments, ensuring the study can be carried out effectively.

3.3.2 Sampling Techniques

The study employed matched, purposive, and random sampling techniques, which are the most commonly used in quasi-experimental studies conducted in educational settings. Matching was operationalized by grouping schools into sets of three referred to as "triples", based on comparable characteristics such as school size, availability of computer laboratories, and geographical proximity. This approach aimed to create comparable groups to reduce selection bias and enhance internal validity.

According to Pournara and Barmby (2019), the weakness of the absence of randomization in quasi-experimental designs is addressed by offering a controlled setting that enhances the reliability of results. Figure 6 summarizes the flow of the sampling procedures.

Following the creation of matched triples, schools within each district were evaluated against inclusion criteria, including flexibility of school administration, multiple Senior Three streams, sufficient computer resources, and proximity. This process yielded three triples in rural districts and two in urban districts. One triple from each district was purposively selected to participate in the study. Within each selected school, two Senior Three classes were randomly assigned to either the experimental group or the control group.

Additionally, 48 learners were purposively selected for follow-up qualitative interviews, ensuring representation across gender, location, achievement level, and instructional strategy. Two mathematics teachers from each participating school were purposively selected to undertake training, lesson delivery, and learner assessment.

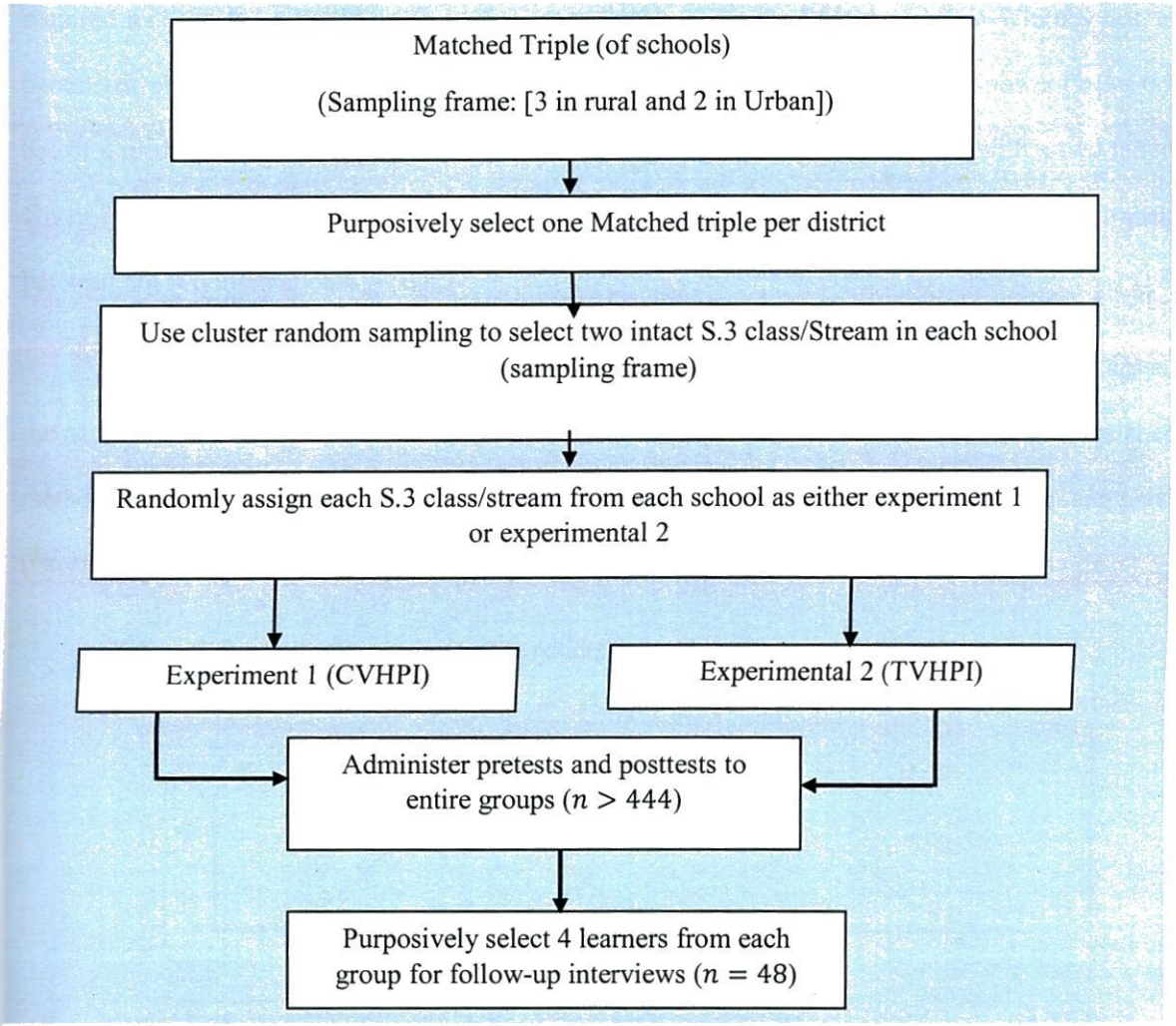


Figure 6: The flow of the sampling procedures followed within the study.

3.3.3 Sample Size

In educational research, sample size justification is based on striking a compromise between statistical power and practical limits, especially in quasi-experimental methods. Due to the complexity of quasi-experimental investigations, the sample size should be determined based on the intended effect size, statistical power analysis, and the reduction of potential confounding variables (Baker, 2018).

In this study, two intact classes of S.3 were selected from the existing streams/intact classes of S.3 in each of the six schools. The G*Power 3.1.9.4 was utilized to determine the minimum sample size

required for this study. Based on an a priori power analysis for the Wilcoxon-Mann-Whitney test, a non-parametric alternative to the independent samples t-test. The study aimed for a 95% power ($1-\beta = 0.95$) to detect a medium effect size (Cohen's $d = 0.5$) at a significance level of $\alpha = 0.05$ with a two-tailed test. Given the quasi-experimental design, an allocation ratio of 0.9 was set to account for slight imbalances between the two instructional groups.

The allocation ratio of approximately 0.9 between the experimental and control groups arose from the natural class sizes of intact streams used in this quasi-experimental design. Such minor imbalances are methodologically acceptable as they do not compromise statistical power or group comparability (Creswell & Creswell, 2018).

Figure 7 shows the screenshot of the results from the software.

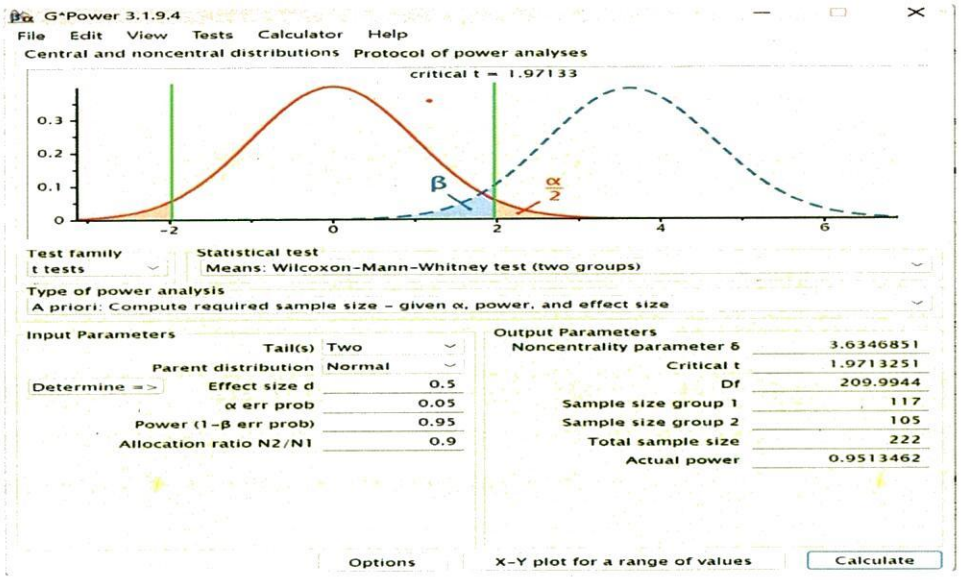


*Figure 7: Showing minimum sample size for each group, obtained from the G*power.*

The computed minimum required sample size was 117 participants for Group 1 and 105 participants for Group 2 per district, yielding a total of 222 participants per district. Since the study was

conducted in two districts, the total required sample size across both districts was 444 participants (234 in Group 1 and 210 in Group 2). However, due to natural classroom settings, the actual sample used in the final analysis included 238 participants in Group 1 and 245 participants in Group 2, resulting in a total of 483 participants. *Table 3* indicates participant count and completion rates for pre-test and post-test by Group.

Table 3: Participants Count and Completion Rates for Pre-Test and Post-Test by Group

School	TVHPI-Group				CVHPI-Group			
	Pretest		Posttest		Pretest		Posttest	
	ATT	ACH	ACH	ATT	ATT	ACH	ACH	ATT
Urban School (U1)	86	76	74	69	67	67	65	56
Urban School (U2)	82	82	79	79	76	73	68	53
Urban School (U3)	48	47	44	42	52	52	45	31
Rural School (R1)	69	60	56	47	74	57	50	49
Rural School (R2)	76	43	43	41	56	41	40	40
Rural School (R3)	23	22	22	18	61	53	41	37
Total Cases	384	330	318	296	386	343	309	266
Attrition	**146**				**141**			
Complete Cases	**238**	**238**	**238**	**238**	**245**	**245**	**245**	**245**

ATT=Attitude Test, ACH=Achievement Test, U=Urban, R=Rural

According to Table 3, the study initially recorded 384 participants in the TVHPI-Group and 386 in the CVHPI-Group across six schools during the pre-test. However, due to incomplete responses, student absenteeism, and attrition, the number of complete cases available for post-test analysis decreased. By post-testing, 330 participants were in the TVHPI-Group and 343 in the CVHPI-Group. Further data cleaning resulted in a final dataset of 238 complete cases in the TVHPI-Group and 245 in the CVHPI-Group, leading to a total of 483 participants for final analysis, with attrition of 146 learners in the

TVHPI group and 141 learners in the CVHPI group. While the drop in sample size slightly deviates from the planned distribution, the final sample still exceeded the required minimum of 444 participants, ensuring sufficient statistical power for detecting meaningful effects.

The sample used for the qualitative phase was purposively selected to ensure the inclusion of diverse learners equally distributed across gender (24 males and 24 females), location (24 urban and 24 rural), achievement level (24 lower achievers and 24 higher achievers), and instructional strategies (24 taught using TVHPI and 24 using CVHPI); maintaining a total of 48 learners.

3.4 Study Setting and Procedures

The study was conducted within the senior three class in secondary schools in rural and urban districts in -western Uganda. The area of geometry that was covered in the study is transformation geometry (The alignment of the Van Hiele Levels and Learning Outcomes for Transformation Geometry in the Uganda lower secondary school curriculum is presented in Ndungo (2024)3).

All participating teachers underwent two training days to prepare them for implementing interventions in their respective classes. The training aimed to equip teachers with the necessary skills to deliver lessons using both the Technology-Enhanced Van Hiele Phased Instruction (TVHPI), which incorporates GeoGebra, and the Conventional Van Hiele Phased Instruction (CVHPI), which is non-technology. It covered the theoretical framework and practical application of the Van Hiele Phases of instruction, focusing on guiding students through different levels of geometric thinking.

Collaborative lesson planning and microteaching were also conducted during the training to ensure consistency in lesson delivery during the intervention. Separate trainings were held for each

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[3] Ndungo, I. (2024). An assessment of the alignment of transformation geometry with Van Hiele levels in Uganda's lower secondary mathematics curriculum. *Turkish Journal of Mathematics Education, 5*(2), 33–41. https://tujme.org/index.php/tujme/article/view/108

district from 8:00 am to 5:00 pm, totaling 18 hours of continuous training. Additionally, support training was conducted by the researcher during the intervention period to help individual teachers effectively implement the intervention. The training content was organized in the teacher training manual developed by the researcher, which can be accessed via the link4.

After the teacher training, pretest assessments for attitude and achievement were administered to S.3 learners in the participating schools to establish baseline attitudes and achievement, allowing for an accurate assessment of the instructional methods' impact over time.

Following this assessment, learners in each school were taught transformation geometry using the two instructional strategies for six weeks. Each group (CVPHI and TVHPI) received four lessons of 40 minutes per week, 24 lessons by the end of the intervention, delivered following the regular school timetable. Before the start of the formal lessons, the experiment 2 group (TVHPI) learners were given a briefing and guidance on using GeoGebra. All of their lessons were conducted in a computer laboratory using GeoGebra.

The same teacher taught both groups to maintain consistency, while the researcher monitored and provided support to teachers during the intervention period to ensure fidelity. The setup for GeoGebra was then installed on all the computers in the school computer laboratories; the teachers used projectors to deliver lessons, while the learners used the computers to perform different transformation tasks, with lessons taking place in the computer laboratories.

In the case of the experiment 1 group (CVHPI), lessons were conducted in standard classrooms without the use of technology. The teacher relied on the chalkboard to demonstrate transformations, while learners used graph books or papers along with mathematical instruments to carry out the tasks. After six weeks of instruction, post-tests were administered to all participants to evaluate changes in their attitudes and Van Hiele levels.

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[4] https://tinyurl.com/3uhkp7rt

One week after the intervention ended, follow-up interviews were held with 48 purposively selected learners to gain qualitative insights into their experiences and comprehension of the geometry content. Each interview lasted between five and ten minutes and was scheduled in the early mornings or evenings to avoid disrupting regular lessons. To provide balanced exposure to technology without compromising the integrity of the instructional approach, learners in the CVHPI group were granted one week of access to GeoGebra after the intervention.

3.5 Data Collection Tools

The data was collected using five data collection tools, yielding five sets: the pre-test and post-test attitudes from the Transformation Geometry Attitude Scale (TGAS1) and TGAS2, respectively; the pretest and post-test achievement from the Van Hiele Geometry Achievement Test (VGAT1) and TGAT2, respectively; and the qualitative data on learners' experiences and understanding of geometry from the Interview Guide. The data collection instruments are attached in the appendix and can also be accessed through the link[5].

3.5.1 Transformation Geometry Attitude Scale (TGAS)

This scale consisted of two similar tools, the TGAS1 and TGAS2, administered at the pre-test and post-test, respectively. These tools consisted of the same items measuring similar constructs, with simple rephrasing to create a slight difference in the appearance of the items, thereby preventing learners from directly transferring their pre-test experiences to the post-test assessment. The researcher developed the scale through a rigorous validation process to ensure its reliability and appropriateness in assessing learners' attitudes toward transformation geometry. The items in the scale were guided by an extensive

[5] https://tinyurl.com/m2nwxaa5

literature review, drawing insights from the works of Avcu (2015), Doz et al. (2022), Uwurukundo et al. (2022), Yea-Ling Tsao & Wei-Shin Tung (2022), Mazana et al. (2018), Opolot-Okurut (2005), and Abdullah and Zakaria (2021).

The original scale comprised 36 items that captured key constructs such as interest and confidence, learning engagement and metacognition, and transformation geometry's perceived relevance and applications. A panel of five experts, including secondary school mathematics teachers and university mathematics lecturers, reviewed the scale for content validity. Following their feedback, minor changes were made to the items' clarity and theoretical alignment.

Following this was a pilot study with 132 S.3 learners from two secondary schools with similar characteristics to those selected for the study. The generated data was used for initial validation and was analyzed by the lead researcher using Excel (CSV) and Python. This analysis yielded a Cronbach's Alpha of 0.85, confirming strong reliability. The Bartlett's Test of Sphericity ($\chi^2 = 891.48$, $p < 0.001$) and the Kaiser Meyer Olkin (KMO) measure of 0.81 confirmed the suitability of the data for exploratory factor analysis. A 12-factor model emerged; nevertheless, a more interpretable 3-factor solution was selected based on theoretical coherence and statistical fit.

The grouping included affective factors related to interests, enjoyment, and confidence, with an internal consistency of $\alpha = 0.80$. The behavioral component consisted of learning engagement, effort, and metacognition, with an internal consistency of $\alpha = 0.77$. The cognitive component consisted of items related to the relevance, application, and importance of transformation geometry, with an internal consistency of $\alpha = 0.85$.

Finally, the original 36 items were reduced to 22 by removing items with low factor loadings (<0.4) or poor item-total correlations. The structure of the final tool, which maintains the initial codes for proper referencing to the original tool, is shown the appendix section. After validating the tool during the pilot study, actual data were collected and analyzed to confirm the structure of this attitude scale.

The interrelationships among the three attitudes were found to be 0.705 between Factor 1 (Affective) and Factor 2 (Behavioral), indicating a strong conceptual alignment between emotional and behavioral components of attitude. Factor 3 (Cognitive) exhibited moderate correlations with both Factor 1 (r = 0.500) and Factor 2 (r = 0.529). This suggests that, though the cognitive dimension is different, it preserves a meaningful association with both emotional and behavioral aspects, reinforcing the structural validity of the attitude scale. The details of the validation process can be found in Ndungo et al. (2025)[6].

3.5.2 The Van Hiele Geometry Achievement Test (VGAT)

The Van Hiele Geometry Achievement Test (VGAT) was developed to assess learners' Van Hiele levels. Two similar tests were set, TGAT1 and TGAT2, to cater for pre-test and post-test, respectively. These tests were of the same structure, assessing questions from the same concepts but numerically different.

Although the learners had not been formally taught transformation geometry as outlined in the current curriculum, the pre-test served as a baseline measure of their initial level of geometric reasoning, as outlined in the Van Hiele framework. The Van Hiele model does not assess prior content knowledge per se, but rather a learner's ability to reason geometrically, starting from the visualization level and progressing toward deductive reasoning. Thus, learners may exhibit varying Van Hiele levels without direct prior instruction based on informal knowledge, previous exposure, or developmental readiness. This approach is consistent with prior studies employing Van Hiele-based interventions.

A panel of five experts in mathematics education and secondary school teachers assessed the test's relevance, clarity, and alignment to the mathematics curriculum. To establish face validity, a pilot

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[6] Ndungo, I., Balimuttajjo, S., & Akugizibwe, E. (2025). Transformation Geometry Attitude Scale (TGAS): Development and Validation for Secondary School Learners in Uganda. *International Journal of Advanced Research, 8(1), 140-150.* https://doi.org/10.37284/ijar.8.1.2861

study was conducted involving thirty S.3 learners randomly selected from two secondary schools with similar characteristics to those participating. The learners' responses to the test items provided feedback on question clarity and difficulty, leading to minor modifications for better comprehension.

Construct validity was ensured through the hierarchical arrangement of questions, with each level progressively building upon prior knowledge. To ensure uniform grading, minimize subjective bias, and guarantee reliability and consistency in marking, the test scripts underwent double marking by other experts in mathematics education.

3.5.3 Assessment of Learners' Geometric Reasoning Using the Transformation Geometry Achievement Test (VGAT)

The TGAT was scored using a weighted rubric: learners received 1 point for correctly answering at least three questions at Level 1, 2 points for Level 2, 4 points for Level 3, and 8 points for Level 4, resulting in a maximum possible score of 15 points, as guided by Vojkuvkova (2012).

However, point totals alone were insufficient for determining a learner's Van Hiele level. Instead, the scoring framework followed a cumulative placement rule, where a learner was placed at a given level n only if they correctly answered at least three items at that level and each preceding level. For instance, a learner who correctly answered three items at Level 1, two at Level 2, and three at Level 3 would receive 5 points but only be placed at Level 1, failing to meet the minimum requirement at Level 2. This rule ensured that learners' placement reflected sequential and stable cognitive development, consistent with Van Hiele's theoretical structure. Figure 8 shows the algorithm followed when placing learners at different Van Hiele levels following the scoring rubric.

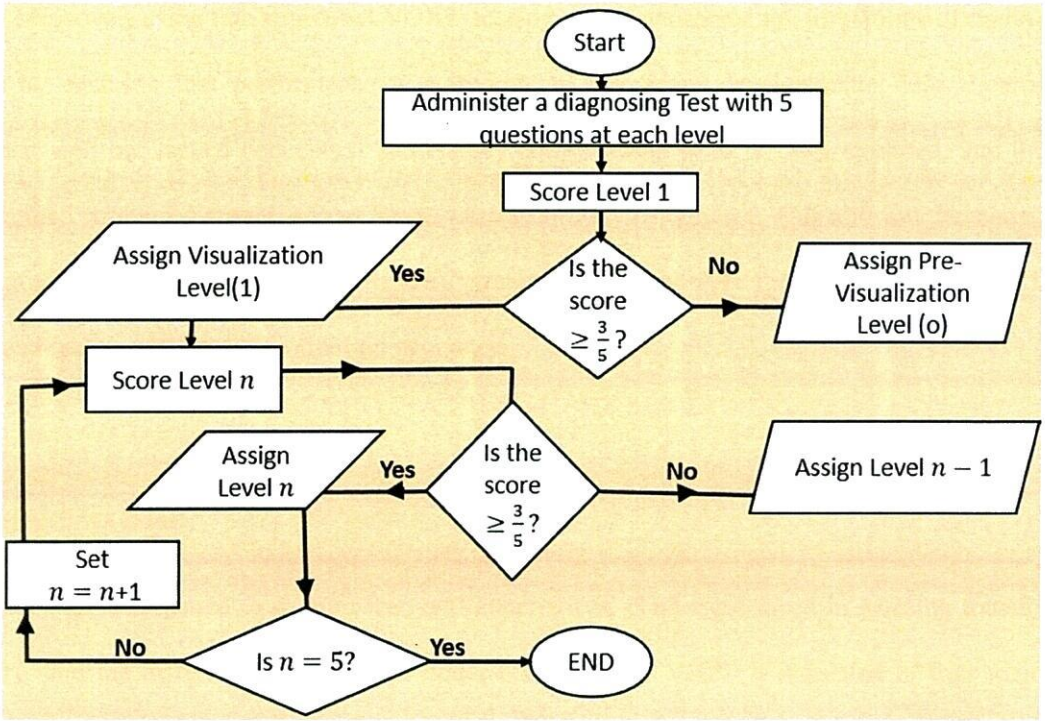


Figure 8: Algorithm for placing learners at different Van Hiele levels

The flowchart illustrates how learners are placed at appropriate Van Hiele levels based on their performance in the Van Hiele Geometry Achievement Test (VGAT). Learners start at Level 0 (pre-visualization) and progress sequentially through Levels 1 to 4, advancing only if they correctly answer at least 3 out of 5 items (≥60%) at each level. Failure at any level stops progression, and the last successfully achieved level is recorded as the learner's placement. This diagnostic process identifies learners' current geometric reasoning stage and guides teachers in providing targeted, level-appropriate instruction.

This scoring approach was justified by the foundational principle of the Van Hiele model: progression through levels is not automatic, but rather occurs through the internal restructuring of reasoning. Each level builds on the previous one, and mastery of a higher level presupposes consistent understanding of the preceding stages. As such, the cumulative placement criterion avoids the misclassification that might occur if learners were allowed to "skip" levels based on isolated success in higher-level items.

Moreover, using this structured VGAT scoring model addressed the limitations of traditional test scoring by ensuring that performance was interpreted through a developmental lens. Learners who performed well but lacked conceptual consistency across levels were not over-credited, and those who demonstrated structured growth across levels were accurately recognized. This allowed the study to make meaningful comparisons between instructional strategies and to analyze the nature and depth of learners' conceptual progression in transformation geometry.

3.5.4 Interview Guide

This guide was formulated to capture learners' experiences, challenges faced in learning transformation geometry, and the different support needs under CVHPI and TVHPI. It consisted of four major open-ended questions, each with probing items, intended to elicit in-depth qualitative responses. The formulation of the questions was guided by the existing literature on mathematics education, instructional strategies, and learners' cognitive development, ensuring the content validity of the tool.

To ensure the clarity of the questions and alignment with the study objectives, two mathematics education experts at the university level and one experienced secondary school teacher were asked to review them, providing feedback that led to minor refinements. The tool was then piloted with a small group of 8 learners to assess question clarity and response depth, and final adjustments were made to the questions. The interview guide is presented in the appendix section.

3.6 Data Analysis

This section presents analysis methods according to the study objectives, starting with the first through the fourth. The data type, the tools and techniques used, and the approach taken to ensure validity and accuracy in the analysis are also clearly outlined.

3.6.1 Overview of the Data Analysis Process

This study generated quantitative and qualitative data that required a mixed-methods approach integrating quantitative and qualitative analysis techniques. The data were collected from the Transformation Geometry Attitude Scale (quantitative), the Van Hiele Geometry Achievement Test (Quantitative), and interviews (Qualitative). Both quantitative and qualitative data were organized using Excel. Simultaneously, Python was utilized for statistical analysis, and advanced Excel tools were employed for coding and thematic analysis of the qualitative data.

The credibility of the findings was ensured through expert and peer review. The data analysis began with preliminary tests for reliability using Cronbach's Alpha and normality using Shapiro-Wilk and Kolmogorov-Smirnov tests, followed by analysis per objective. The detailed analysis methods for each objective are presented in Subsections 3.6.2 through 3.6.6.

3.6.2 Objective 1: Comparing the Effect of TVHPI and CVHPI on Learners' Attitudes

The first objective aimed to establish the differences in learners' attitudes from pre-test to post-test and the attitude change between the two instructional strategies. The analysis utilizes descriptive and inferential statistics to address the two hypotheses under the objective. The first hypothesis (within-group) assumed no significant shift in learners' attitudes before and after instruction in both groups (H_{01}), with its alternative suggesting a significant shift in both groups after instruction (H_{a1}). The second hypothesis (between-group) posits no significant difference in attitude shifts between TVHPI and CVHPI learners (H_{02}), with its alternative suggesting a significant difference in attitude shift between TVHPI and CVHPI (H_{a2}).

Due to the data's non-normal distribution, the first hypothesis (H_{01}) was tested using the Wilcoxon Signed-Rank Test. In contrast, the Mann-Whitney U test tested the second hypothesis (H_{02}). A

difference-in-differences (DiD) analysis using an ordinary least squares (OLS) regression model was conducted to separate the instructional effect over time and control for pre-existing variations and post-interventional attitudes. The effect size of attitudinal shifts and the strength of the observed differences were assessed using the Rank-Biserial Correlation (r_B). Figure 9 illustrates the data summary, including the relevant tests to address this objective.

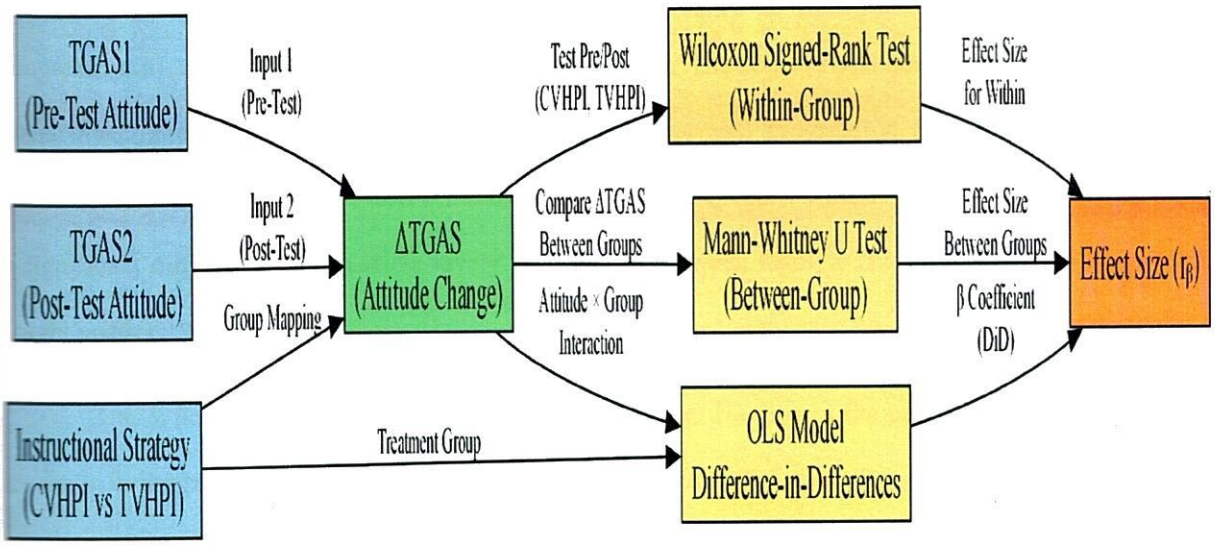


Figure 9: Data mapping strategy showing how the first objective was achieved

3.6.3 Objective 2: Comparing the Effect of TVHPI and CVHPI on Learners' Progression through Van Hiele Levels

Similar to the first objective, both descriptive and inferential statistical analyses were utilized to test the two hypotheses under this objective. The third hypothesis (within-group) (H_{03}) proposed that learners exposed to TVHPI or CVHPI would not exhibit significant progression in Van Hiele levels from pre-test to post-test, with its alternative (H_{a3}) positing that each instructional strategy would lead to significant progression in Van Hiele levels from pre-test to post-test. The fourth hypothesis (between-group) (H_{04}) stated that there would be no significant difference in learners' progression through Van Hiele levels between learners taught using TVHPI and those taught using CVHPI, with its alternative (H_{a4}) suggesting a significant difference in Van Hiele levels' progression between learners taught using TVHPI and those taught using CVHPI.

The Wilcoxon Signed-Rank Test was utilized to test H_{03} due to the ordinal nature of Van Hiele levels. In contrast, the Generalized Ordered Logit (GOL) model was applied to test H_{04} rather than relying on traditional nonparametric tests such as the Mann–Whitney U test, because the GOL model accommodates the ordinal nature of the dependent variable while allowing for partial proportional odds when the assumption of proportionality is violated. Unlike the Mann–Whitney U test, which only compares distributions between two independent groups without controlling for covariates, the GOL model enables the inclusion of multiple predictors, provides adjusted estimates, and yields interpretable odds ratios across response categories. This makes it more suitable for modeling ordered outcomes in multivariate contexts and evaluating the likelihood of learners transitioning to higher Van Hiele levels under TVHPI relative to the conventional variant.

Furthermore, a difference-in-differences (DiD) analysis was performed within the GOL model framework to evaluate the effect of the interaction between instructional strategies and time. The significance of this approach was to isolate the net effect of instructional strategies on learners' progression by controlling for pre-existing differences in Van Hiele levels before the intervention.

Lastly, Odds Ratios (ORs) were computed from the GOL model to provide insight into how the likelihood of learners moving to a higher Van Hiele level changed depending on the instructional strategy used. Figure 10 illustrates the data-mapping diagram, which addresses the second objective.

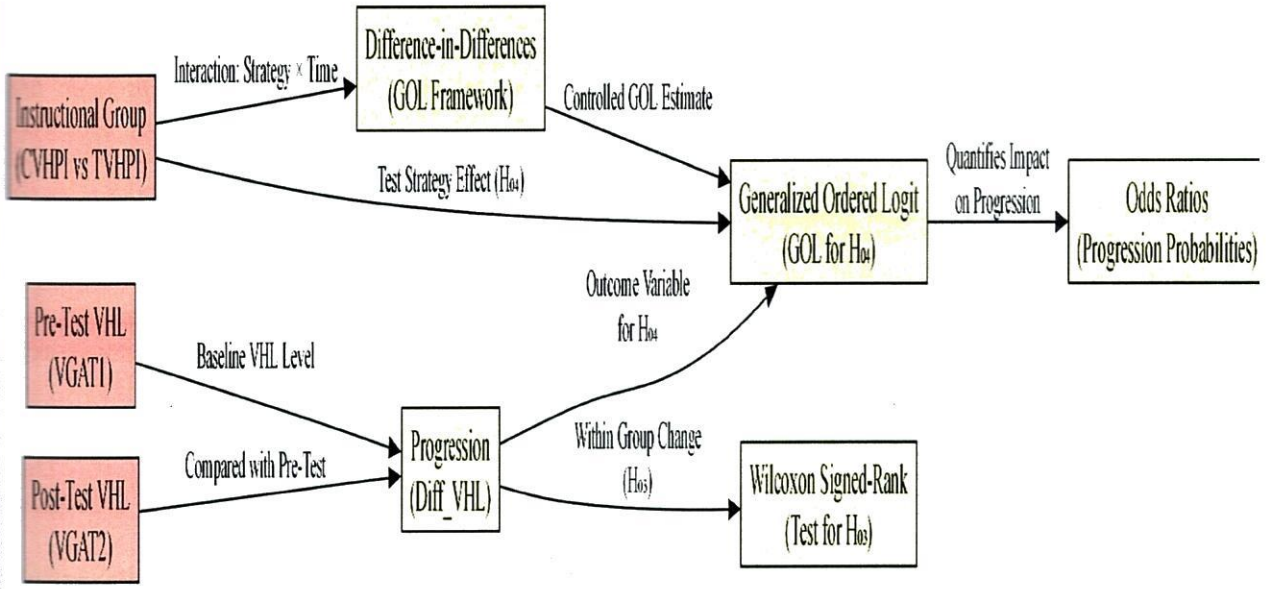


Figure 10: Data mapping strategy showing how the second objective was achieved

3.6.4 Objective 3: Predictive and Mediating Role of Learners' Attitudes in their Progression through Van Hiele Levels

In this objective, the analysis builds on the first and the second objectives by investigating the predictive and mediating role of learners' attitudes in their progression through Van Hiele levels of geometric reasoning across the two instructional strategies. Two hypotheses were tested: the fifth hypothesis (H05) focuses on whether learners' attitudes significantly predict their geometric reasoning levels, and the sixth hypothesis (H06) focuses on whether attitude changes mediate the relationship between instructional strategy and learners' progression through Van Hiele levels.

To address the H_{05}, three Generalized Ordered Logit models were estimated. The ordered categorical nature of the Van Hiele levels necessitated using GOL, rather than traditional correlation-based approaches such as Spearman's rho. The first and second models tested whether pre-test attitudes (TGAS1) significantly predicted learners' initial Van Hiele levels (Pre_VHL) and post-test Van Hiele levels (Post_VHL), respectively. The third model examined whether post-test attitudes (TGAS2) predicted post-test Van Hiele levels (Post_VHL), with the instructional group (TVHPI or CVHPI) included as a control variable in all models to account for baseline differences in learning conditions. Another GOL model was also constructed to determine whether learners' initial Van Hiele levels inform their final levels ($Pre{-}VHL{\rightarrow}Post{-}VHL$).

Similarly, the sixth hypothesis (H_{06}), which evaluates the mediating role of attitude shifts (ΔTGAS) in the relationship between instructional strategies and learners' progression through the Van Hiele levels, was tested using three Generalized Ordered Logit models. All models treated Diff_VHL (Post_VHL – Pre_VHL) as the ordinal dependent variable. The first model analyzed the direct effect of instructional strategy alone. The second model incorporated attitude shift as a predictor to examine its potential mediating effect. The third model included an interaction term between instructional strategy and ΔTGAS to test for moderated mediation. Since GOL does not formally estimate indirect effects, a

complementary bootstrapped mediation analysis was conducted using 5000 resamples. This non-parametric approach assessed whether attitude shift significantly mediated the relationship between instructional strategy and Diff_VHL.

Figure 11 illustrates how learners' geometry attitudes and instructional strategies influence their progression through Van Hiele levels.

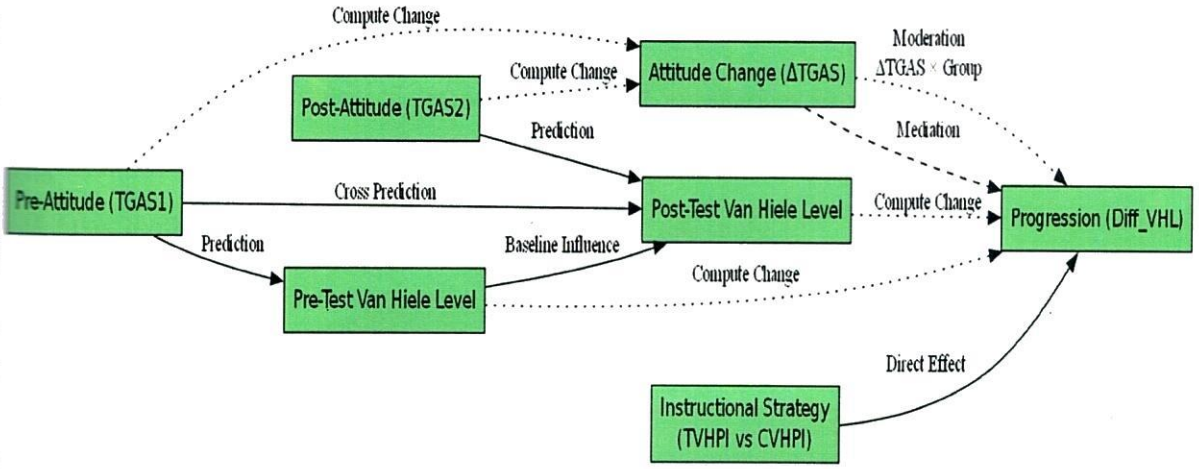


Figure 11: Data mapping diagram illustrating how the third objective was achieved.

The diagram in Figure 11 illustrates the analytical framework for the third objective, showing how learners' attitudes (pre, post, and change) and instructional strategy influence progression through the Van Hiele levels. Solid arrows indicate direct predictive paths, the dashed arrow represents mediation by attitude change, and the dotted arrow shows change computation and the moderation effect (ΔTGAS × Group), examining whether the impact of attitude change varies by instructional method.

3.6.6 Objective 4: Examining how Learners' Experiences and Understanding of Geometry Differ between the two Instructional Strategies.

To address Objective 4, which investigates differences in learners' experiences and understanding of geometry between TVHPI and CVHPI, a thematic analysis was carried out on the post-intervention interview data. This analysis followed a structured six-step process described by Lapolla (2020) and

Meyer & Avery (2009), as outlined in Figure 9. Using this framework allowed for a systematic examination of how students experienced and understood geometry under the two instructional approaches, highlighting nuanced insights into both teaching methods.

To support this interpretive approach, descriptive content analysis techniques were embedded using Excel pivot tables to quantify the frequency of codes across themes and instructional strategies. This integration enabled both depth of interpretation and breadth of comparison, allowing for an understanding of how instructional strategies influenced learners' conceptual experiences in geometry. Excel was used for qualitative coding due to its accessibility and flexibility, while acknowledging that tools such as NVivo and Atlas. ti offer advanced features beyond the scope of this study.

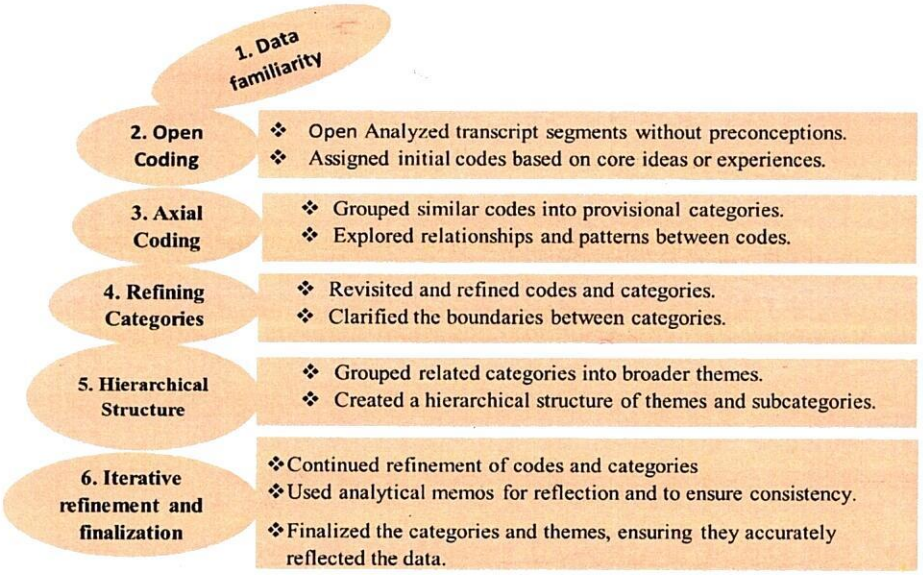


Figure 12: The coding process for thematic analysis of interview data.

Based on the steps outlined in Figure 12, the interview transcripts were read multiple times to gain an in-depth understanding of the data. The data were then segmented, each treated as a unique data item. All segments were transferred into Excel, and a learner's unique identifier was assigned for easy reference. Similar codes were then grouped into categories, followed by a revision of the categories to refine them based on theoretical understanding. Categories were grouped into broader themes, creating a

hierarchical structure. Theme refinement continued until they accurately reflected the data and followed the theoretical understanding of instruction and geometry education.

Once the coding framework was finalized, Excel functions such as sorting, conditional formatting, and concatenation were employed to organize and link the coded segments with learner identifiers. Additionally, pivot tables and charts were used to quantify the frequency of codes under each category and theme, allowing for an integrated comparison across instructional strategies. This quantitative dimension of the qualitative data enhanced the credibility of the thematic narratives developed. The choice of Excel for qualitative analysis over established qualitative data analysis tools like NVivo, ATLAS.ti, and MAXQDA was due to its accessibility, ease of use, and ability to organize and categorize qualitative data in a well-structured way suitable for report writing (Bree & Gallagher, 2016; Eliot, 2011; Garrett, 2015; Lapolla, 2020; Meyer & Avery, 2009; Ose, 2016).

3.7 Ethical Considerations

The ethical guiding principle ensuring protection, confidentiality, and fairness of all participants was followed during the three stages of this study: the pre-intervention stage, the intervention stage, and the post-intervention stage.

The study was approved by the Mbarara University of Science and Technology (MUST) Research Ethics Committee (REC) under approval numbers MUST-2024-1519 and the Uganda National Council for Science and Technology (UNCST) under approval number SS2857ES.

Confidentiality was ensured, and all data were stored securely on password-protected devices accessible only to the researcher. Aggregated results were reported without identifying individual students, teachers, or schools. Informed consent was obtained through a two-step process: first, permission was sought from district education authorities and school administrations, and then written consent was obtained from teachers and students aged 18 and above, alongside parental or guardian

consent for minors. While written approvals were obtained from the city and district education offices, and schools, one urban school, head teacher gave verbal consent but declined to provide written approval despite follow-up. These approvals were documented in field notes, and the school' full participation confirmed consent

Ethical fairness was maintained during the six-week instructional period by ensuring that the same teacher taught the TVHPI and CVHPI groups, using lessons aligned with the official secondary school curriculum. The intervention was integrated into the regular school program, and tests and interviews were scheduled in consultation with the school administration to minimize disruptions to school programs and learners' activities.

After the intervention, ethical fairness in technological exposure was ensured by granting CVHPI learners' access to GeoGebra for one week, allowing them to experience the technological tools used by their TVHPI counterparts. All collected data were stored on a password-protected device, anonymized, cleaned, and organized in Excel for analysis. The lead researcher conducted all the analyses to validate the results, ensuring everything was done reliably and correctly. At the same time, the two research experts/promoters independently reviewed the Excel and the Python codes to ensure fidelity.

CHAPTER FOUR

RESULTS AND DISCUSSION

This chapter presents the study's findings, organized according to the research objectives. For each objective, the results are first presented, followed by a discussion interpreting these findings about existing literature and theory, and concluding with a summary. Section 4.1 addresses the first objective by illustrating how learners' attitudes toward transformation geometry changed over time and across the two instructional strategies. Section 4.2 examines learners' progression through the Van Hiele levels across the two instructional strategies, fulfilling the second objective. The third objective, which investigates the predictive and mediating role of attitudes in learners' progression through Van Hiele levels, is presented and discussed in Section 4.3. Finally, Section 4.4 provides a qualitative analysis of learners' experiences and understanding of transformation geometry within the two instructional strategies.

4.1 Effect of Instructional Strategies on Learners' Geometry Attitudes

This section presents the results for the first objective, which examined how learners' attitudes toward transformation geometry shifted from pretest to post-test within the two instructional strategies: TVHPI and its conventional counterpart, CVHPI. The study tested two null hypotheses:

First, that neither instructional strategy significantly improves learners' attitudes toward transformation geometry from pretest to post-test (H_{01}); and second, that there is no significant difference in the shifts in learners' attitudes between those taught using TVHPI and those taught using CVHPI (H_{02}). The presentation begins with descriptive statistics for preliminary analysis, followed by inferential

statistics for hypothesis testing. The section concludes with a discussion of the findings and a summary. A manuscript from this objective was published and is cited or accessed as in the footnote[7].

4.1.1 Descriptive Statistics of Attitude Shifts

The descriptive statistics capture the reliability test in Table 4, the normality test in Table 5, participants' demographics in Table 6 and Figure 13. It also includes the mean and standard deviation of attitude items in Table 7, and components in Table 8 , and the correlations between attitude components in Figure 14. They culminate with a hypothesized model for the relationships between instructional strategies and attitude in Figure 15, paving the way for inferential analysis.

4.1.1.1 Reliability Test

The reliability test used Cronbach's Alpha to assess the internal consistency of the data instrument used to measure the learners' attitudes toward transformation geometry. Since attitudes are multidimensional, separate tests were done for each dimension: affective, behavioral, and cognitive. The results are indicated in Table 4.

Table 4: Cronbach's Alpha reliability values for each attitude component and overall attitude

Component test	$\alpha -$ **value**
Affective component Cronbach's Alpha	0.7578
Behavioral component Cronbach's Alpha	0.7484
Cognitive Component Cronbach's Alpha	0.8032

[7] Ndungo, I., Balimuttajjo, S., Akugizibwe, E., Muwonge, M.C., (2025). Modeling Learners' Attitudinal Shifts in Transformation Geometry under Technology-Enhanced and Conventional Van Hiele Phased Instruction. *African Journal of Research in Mathematics, Science and Technology Education.* https://doi.org/10.1080/18117295.2025.2529124.

Overall-Attitude-Score Cronbach's Alpha	0.8932

The results in Table 4 indicate the Cronbach's Alpha values for the three components of attitudes and the overall attitude. The affective and behavioral components demonstrated acceptable reliability, while the cognitive and overall attitude exhibited good reliability, confirming that the instrument is statistically reliable.

4.1.1.2 Normality Test

A normality test was conducted on the differences between pretest and post-test attitude scores for each component and the overall attitude score, determining the appropriate statistical methods for analyzing the effect of instructional strategies on learners' attitudes toward transformation geometry. The test aimed to assess whether the differences approximated normality, allowing for the use of parametric tests (the independent t-test and paired t-test), or whether non-normality was present, necessitating the use of non-parametric tests (the Mann-Whitney U test and the Wilcoxon signed-rank test). The results of the normality test are summarized in Table 5.

Table 5: Normality Results for Differences in Attitudes

Variable	S-W Statistic	S-W p-value	K-S Statistic	K-S p-value
Affective-Score (Diff)	0.9913	0.0063	0.0842	0.0020
Behavioral-Score (Diff)	0.9913	0.006	0.0672	0.0241
Cognitive-Score (Diff)	0.9800	<0.001	0.1024	<0.001
Overall-Attitude-Score (Diff)	0.9814	<0.001	0.0539	0.1160

The normality test results in Table 5 indicate that the p-values of the Shapiro-Wilk test and the Kolmogorov-Smirnov test for Affective-scores (Diff), Behavioral-Score (Diff), and Cognitive-score (Diff) are all less than 0.05, suggesting that the differences in the scores for these components deviate from normality. However, for the Overall attitude score (Diff), there are mixed results, with the Shapiro-Wilk test suggesting deviation from normality ($p < 0.001$) and the Kolmogorov-Smirnov test not rejecting normality ($p = 0.1160$), indicating that the difference in overall attitude scores is approximately normally distributed.

With this, since the other components did not meet the normality test and to ensure consistency in the statistical interpretations, non-parametric tests, specifically the Mann-Whitney U-test and Wilcoxon Signed Rank test, were used instead of the independent samples t-test and paired t-test, respectively.

4.1.1.3 Participants' Demographics

Participant demographics were collected to describe the study sample and confirm baseline comparability between experimental groups. Although no subgroup analyses by gender or school location were conducted, ensuring that the groups were demographically similar was critical to support internal validity and to minimize potential confounding effects in this quasi-experimental design. Table 6 illustrates the demographic characteristics of the participants, specifically their gender (male, Female), school location (rural, urban), and instructional group (TVHPI, CVHPI).

Table 6: Demographic Distribution of Participants

Category	Count	Percentage
CVHPI	245	50.7
TVHPI	238	49.3
Female	246	50.9
Male	237	49.1
Urban	250	51.8
Rural	233	48.2

The distribution of the participants' demographic characteristics in Table 6 indicates that the sample was almost evenly split between the two instructional groups, with CVHPI taking a slightly higher proportion of learners, 50.7%, compared to the TVHPI group, which had 49.3%. As for gender, 50.9 % were female, while 49.1% were male, reflecting a fairly equal gender distribution. The proportion of learners from rural schools was 51.8%, slightly higher than that of learners from urban schools of 48.2%.

The demographic distribution of the participating learners was generally nearly balanced, ensuring that the differences in learners' attitudes were likely a result of instructional strategies rather than demographic biases. Likewise, the participants' age distribution is presented in Figure 13

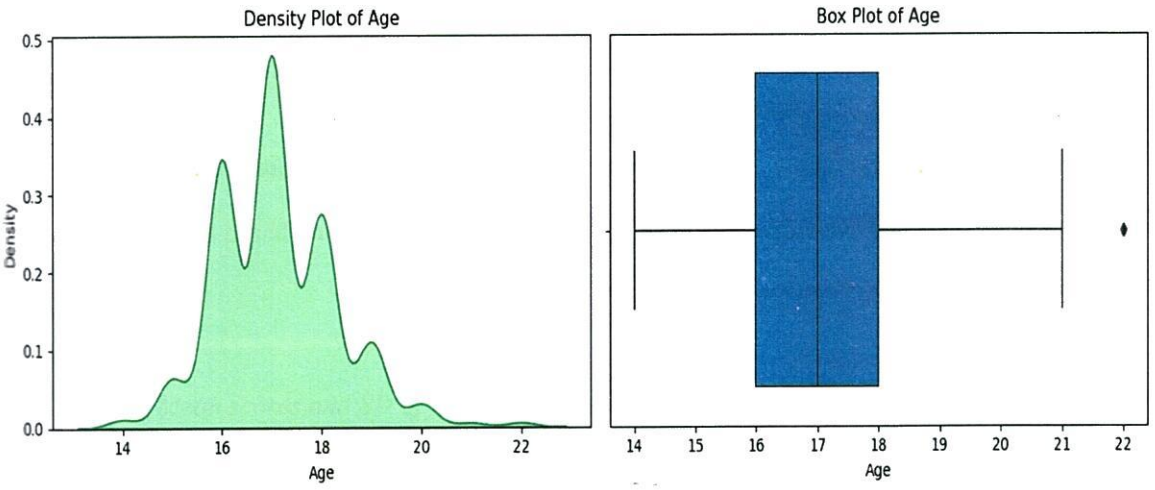


Figure 13: Age distribution of participants using a KDE plot and a boxplot

On the left of Figure 13 is the Kernel Density Estimate (KDE), and on the right is the boxplot that presents the age distribution of participants. From the KDE plot, it is evident that most participants are between 16 and 18 years old, with the model age at around 17 years. The slightly right-skewed shape suggests that a few older participants were present, though the majority were within the expected age range for secondary school learners. Moreover, there were very few participants aged below 14 or above 22. These results reinforce the concentration of ages around the secondary school level. The boxplot reinforces this, indicating that learners aged 22 years are marked as outliers, while most learners are spread from 15 to 20 years, as represented by the whiskers.

4.1.1.4 Comparison of Attitude Components across Instructional Strategies

The learners' attitudes towards transformation geometry were measured using a Likert scale, level 5, where learners were asked to rate the attitude statements from 1 (strongly disagree) to 5 (strongly agree). The mean and standard deviation (SD) for each attitude item were determined by averaging all the learners' responses for that particular item. This was done for both instructional strategies and for both the

pretest and posttest to provide a preliminary trend on how attitudes changed from pretest to posttest. Table 7 presents the means and SD per item of learners' attitudes.

Building on this, items corresponding to each attitude component were combined to yield composite attitudes based on the three attitude components. The component-level summary is in Table 8.

Table 7: Item Mean scores and SD of learners' attitudes toward transformation geometry

Items	TVHPI				CVHPI			
	Pre-Test		Post-Test		Pre-Test		Post-Test	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Interest in transformation geometry	3.10	1.15	3.89	0.91	2.86	1.19	3.65	1.07
Attention is drawn to transformations	3.16	1.15	4.00	0.86	2.78	1.15	3.64	1.13
Excitement about transformations	2.44	1.40	3.58	1.13	2.56	1.28	2.78	1.28
Joy in discovering shape connections	2.84	1.31	3.96	1.00	2.72	1.09	3.53	1.31
Motivation to complete tasks	3.11	1.19	4.05	0.87	2.98	1.17	3.51	1.13
Confidence in understanding	3.21	1.21	4.00	1.01	2.96	1.19	3.61	1.06
Self-assurance in solving problems	3.14	1.24	3.98	0.93	3.11	1.19	3.40	1.05
Seeking additional information	3.26	1.20	3.96	0.92	3.02	1.22	3.63	0.95
Curiosity about transformations	2.96	1.27	3.85	1.03	2.57	1.15	3.44	1.04
Anticipation for transformation lessons	3.37	1.15	3.98	0.94	3.03	1.14	3.44	1.04
Applying transformations in real life	2.67	1.21	3.74	1.03	2.61	1.15	3.45	1.06
Varied activities enhance learning	3.29	1.17	3.92	0.96	2.94	1.22	3.51	1.01
Exploring advanced concepts	2.76	1.27	3.71	1.03	2.71	1.17	3.33	1.02
Confidence in explaining concepts	2.53	1.41	3.34	1.07	2.42	1.21	2.90	1.25
Independence in learning new topics	2.74	1.25	3.61	1.04	2.70	1.19	3.31	1.13
Importance of transformations for the future	3.30	1.24	4.14	0.95	3.04	1.11	3.74	1.08
Connecting transformations to the real	2.84	1.20	3.77	1.05	2.92	1.20	3.49	1.02

Items	TVHPI				CVHPI			
	Pre-Test		Post-Test		Pre-Test		Post-Test	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
world								
Use of transformations beyond mathematics	3.34	1.11	4.05	0.94	3.02	1.08	3.61	0.99
Applications of transformations beyond class	3.24	1.14	3.89	1.12	3.15	1.17	3.73	1.01
Enhancing critical thinking through	3.22	1.09	3.83	1.05	3.22	1.17	3.62	0.98
Contribution of transformations to education	3.18	1.12	3.83	1.05	3.18	1.13	3.60	1.01
Value of transformations across disciplines	3.07	1.18	3.90	1.05	3.15	1.16	3.73	0.94

The mean scores and standard deviations for individual attitude items across the TVHPI and CVHPI groups, comparing both pre-test and post-test results, are presented in Table 7. These results provide insights into how learners' attitudes toward transformation geometry shifted following intervention. For example, it is noted that the post-test mean scores are considerably higher than the pre-test mean scores in all items and instructional strategies, suggesting an improvement in attitudes from pre-test to post-test across all instructional groups. Given that the trend is reflected in both instruction groups, both strategies positively influenced learners' attitudes. Although the comparisons in Table 7 do not account for baseline attitudes, the post-test mean scores indicate that the TVHPI group demonstrated higher scores in most items compared to its conventional variant.

The stable standard deviations in most items suggest reduced variability in the post-test results, which may imply that the instructional interventions helped align learners' perceptions. Notably, items related to interest, confidence, and real-world applications of transformation geometry had better mean scores in the TVHPI than in the CVHPI, possibly suggesting that GeoGebra's interactive and visual nature enhanced learner engagement and made abstract concepts more tangible and relatable. Similarly, notable

improvements were observed in areas related to effort and motivation in the CVHPI compared to the TVHPI group, indicating that the structured and hands-on nature of conventional instruction may have supported persistence and task-focused behaviors among learners.

Table 8: Component Mean scores and SD of learners' attitudes toward transformation geometry

Components	CVHPI				TVHPI			
	Pre-Test		Post-Test		Pre-Test		Post-Test	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Affective	2.85	1.20	3.42	1.07	3.02	1.23	3.86	0.96
Behavioral	2.75	1.16	3.40	1.13	2.93	1.25	3.82	0.99
Cognitive	3.10	1.15	3.65	1.00	3.17	1.15	3.92	1.03

From Table 8, the mean scores for all three components, Affective, Behavioral, and Cognitive, increased from pre-test to post-test across both instructional strategies. Although the current comparison does not adjust for baseline values, the TVHPI group consistently demonstrated higher post-test means than the CVHPI group across all components. Specifically, the Affective component showed a mean of 3.86 (SD = 0.96) in the TVHPI group, compared to 3.42 (SD = 1.07) in the CVHPI group.

The Behavioral component followed a similar pattern, with a post-test mean of 3.82 (SD = 1.00) in the TVHPI group and 3.40 (SD = 1.13) in the CVHPI group. Lastly, the Cognitive component recorded a post-test mean of 3.92 in the TVHPI group, higher than the 3.65 observed in the CVHPI group. These patterns reinforce the tendencies observed at the item level. They may point to the potential influence of GeoGebra's interactive and visual affordances in enhancing learners' attitudes toward transformation geometry under the TVHPI approach. These changes are visually presented in Figure 14, highlighting the differences in learners' attitudes from pre-test to post-test scores in the two instructional strategies for easier pattern observations.

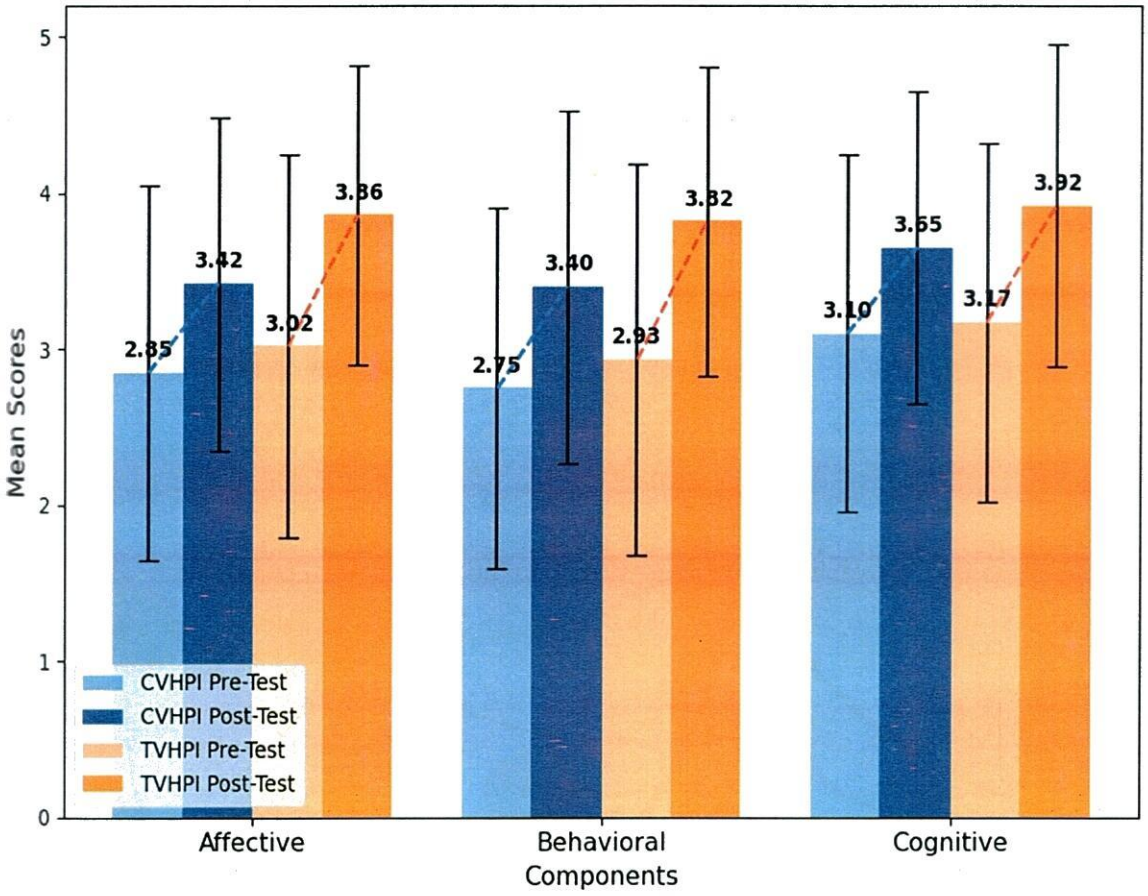


Figure 14: Comparison of Mean Scores across Instructional Strategies

Figure 14 illustrates a visual representation to supplement the results in Table 8, comparing the mean scores of the three components across the two instructional strategies before and after the intervention. The visual highlights that TVHPI demonstrates greater gains than its conventional variant. The trend line emphasizes the magnitude of the change, while the error bars indicate the variability. This visual graph highlights that technology-enhanced Van Hiele phased instruction substantially impacts learners' attitudes toward transformation geometry more than conventional Van Hiele phased instruction.

4.1.1.5 Conceptual Model for the Relationship between Instructional Strategies and Attitudes

Following the descriptive statistics, variations in learners' attitudes across the two instructional strategies and changes over time are suggested. These attitudinal trends require additional analyses to determine the statistical significance of the observed differences. A hypothesized model illustrating the expected relationships between instructional strategies, time, attitude components, and overall attitudes towards transformation geometry is presented in Figure 15.

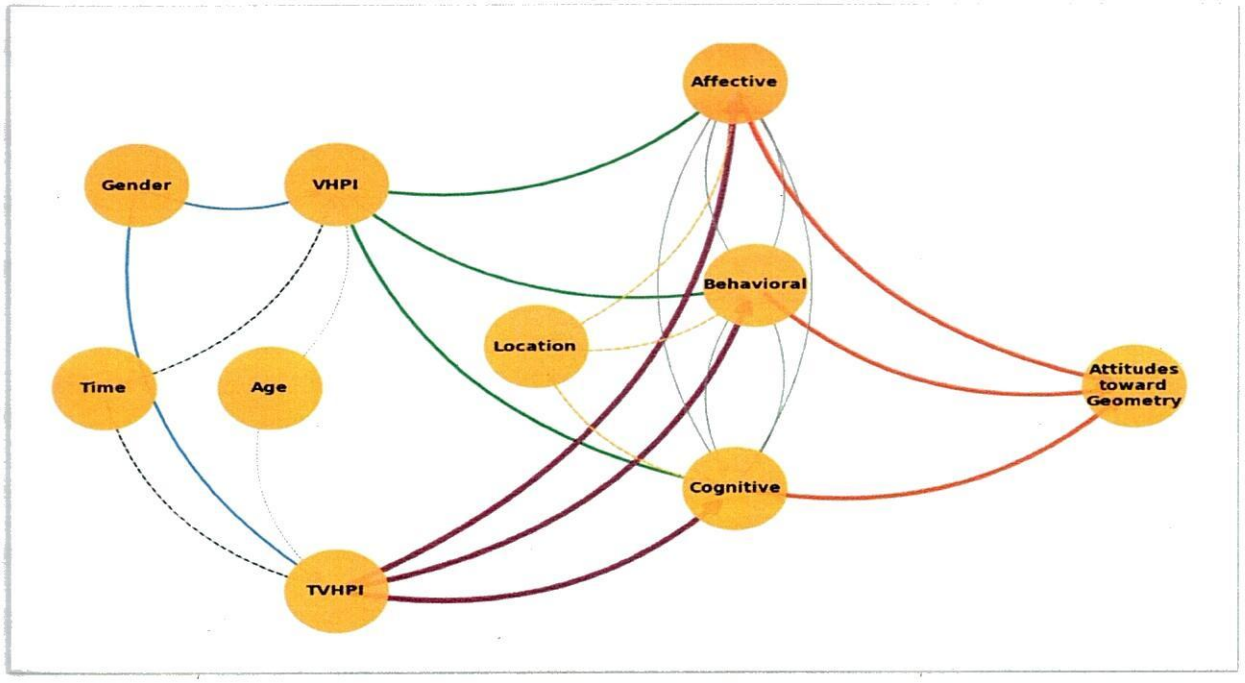


Figure 15: Hypothesized model for the relationships between instructional strategies and attitude

The model in Figure 15 hypothesizes that CVHPI and TVHPI influence learners' affective, behavioral, and cognitive attitudes, with time expected to show a shift in attitudes within each strategy. While the inferential analysis focuses solely on instructional strategy and time, the model also incorporates gender, location, and age to represent the broader learning context. The demographic factors have been included in the model to acknowledge that learning experiences do not occur in isolation. These factors are not directly tested, and their effect was controlled by ensuring balanced representation.

4.1.2 Instructional Strategies and Attitude Shifts (within-group analysis)

In this subsection, the first hypothesis (H_{01}) is tested using the Wilcoxon Signed-Rank Test. The hypothesis focuses on whether TVHPI and CVHPI significantly improve learners' attitudes toward transformation geometry from pre-test to post-test.

4.1.2.1 Respondents' Attitude Change by Instructional Strategy

In this subsection, the results of the analysis of respondents' attitude change toward transformation geometry, after instruction through either CVHPI or TVHPI, are presented. The data for this analysis are summarized in Figure 16.

In Figure 16, the x-axis represents the categories of attitude change: positive, negative, and no change for each instructional group, while the y-axis represents the count of respondents. All instructional groups demonstrated a high number of learners under the positive change category, indicating that in both groups, the majority of learners experienced positive change in their attitudes towards transformation geometry after the intervention.

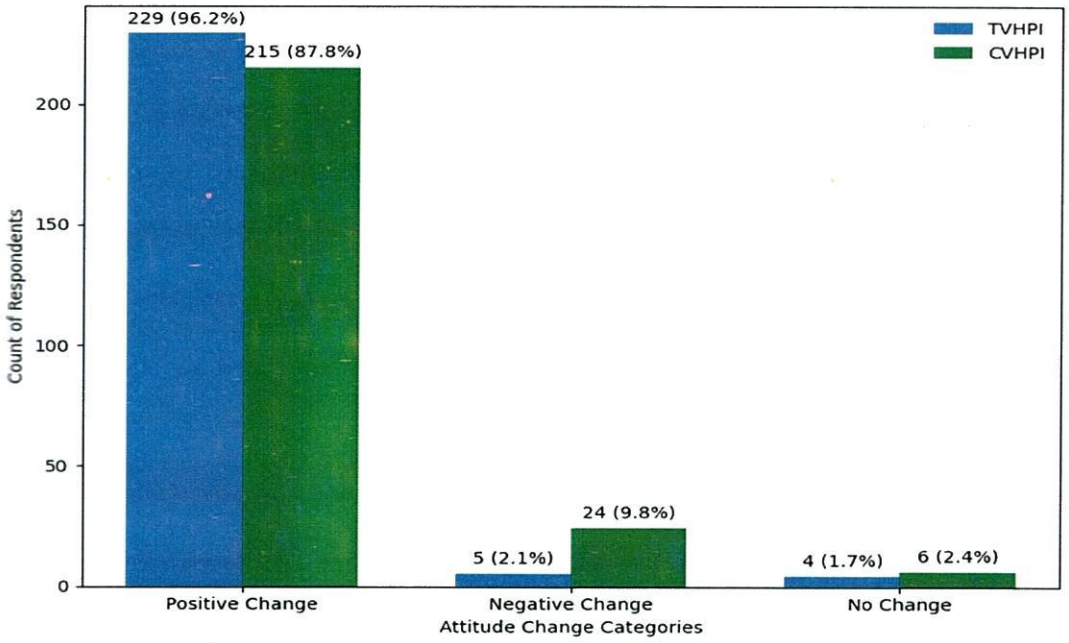


Figure 16: Respondents' Attitude Change by Instructional Strategy

However, the TVHPI group showed a higher percentage (96.2%) of respondents under this category compared to the CVHPI group (87.8%), highlighting that the technology-based instructional strategy had a comparative advantage in positively shifting learners' attitudes over its conventional variant, mainly due to its interactive and visual abilities. Moreover, both groups had learners who experienced negative and no changes, with the CVHPI group having slightly more learners in these categories. Generally, the graph shows that the TVHPI group was more effective in promoting learners' attitudes with higher positive, fewer negative shifts and a small proportion of learners showed no change compared to the CVHPI group.

4.1.2.2 Wilcoxon Signed-Rank Test Results for TVHPI and CVHPI

The Wilcoxon Signed-Rank Test was conducted to confirm whether there was a significant change in learners' attitudes toward transformation geometry before and after the intervention in each instructional group, TVHPI and CVHPI. The results are summarized in Table 9.

Table 9: Wilcoxon Signed-Rank Test Results for TVHPI and CVHPI

Instructional Strategy	Wilcoxon Statistic	P-Values	(r_β)
TVHPI	909.5	3.80e-36	0.998
CVHPI	49.0	7.11e-40	0.970

Table 9 presents a comparative overview of the impact of two instructional strategies, TVHPI and CVHPI, on learners' attitudes. The Wilcoxon Signed-Rank Test was employed to determine whether the observed differences were statistically significant, allowing for a non-parametric assessment of changes in attitudes within each instructional group. This approach provided a robust comparison by accounting for the paired nature of the pre- and post-intervention data while avoiding assumptions about the normality of the distribution.

The Wilcoxon test yielded very small values of p for both instructional strategies ($p = 3.80e-36$ *and* $p = 7.11e-40$). With these smaller values of p, it is concluded that the changes are highly significant, providing evidence to reject the null hypothesis (H_{01}) and support the alternative hypothesis. The results, therefore, confirm that both TVHPI and CVHPI significantly improved learners' attitudes towards transformation geometry from pretest to post-test. The very small p-values reflect strong statistical significance, partly due to the large sample size (N = 483). Practical significance was verified using effect sizes, which confirmed that the differences were educationally meaningful, not merely a result of sample size sensitivity. The greater value of r_β for TVHPI (r_β = 0.998) compared to CVHPI (r_β =

0.970) shows that TVHPI is more effective than CVHPI, suggesting that TVHPI may offer a higher benefit in shifting learners' attitudes compared to its conventional variant.

4.1.3 Comparative analysis of Attitude shifts between Instructional Strategies

This subsection presents the results testing the second hypothesis (H_{02}) to determine whether there is a significant difference in learners' attitude shifts between the TVHPI and CVHPI groups. A series of statistical analyses were used to test the hypothesis, including the Mann-Whitney U test, Cliff's delta effect size, Rank-Biserial Correlation, Ordinary Least Squares regression, and the difference in difference method.

4.1.3.1 Mann-Whitney U Test for comparing shifts between instructional strategies

Table 10 summarizes the Mann-Whitney U Test and Cliff's Delta Effect Size for comparing TVHPI and CVHPI instructional strategies.

Table 10: Mann-Whitney U Test and Cliff's Delta Effect Size for TVHPI vs. CVHPI

Comparison	Mann-Whitney U Statistic	p-value	Cliff's Delta Effect Size
TVHPI vs. CVHPI	37247.5	6.5445e-08	0.2776

The results in, the Mann-Whitney U test revealed a significant difference between TVHPI and CVHPI ($U = 37{,}247.5$, $p < 0.001$). The effect size, measured by Cliff's Delta ($\delta = 0.278$), indicates a small-to-moderate advantage in favor of the TVHPI group. Therefore, it is suggested that learners taught using TVHPI demonstrated significantly greater positive attitude shifts compared to those taught using

traditional CVHPI. The results therefore support the alternative hypothesis (H_{a2}) against the null hypothesis (H_{02}). To further confirm this results, a DiD graph using the mean attitudes was fitted, to compare the trend of the attitudinal shifts between the two instructional strategies as explained in the subsection that follows.

4.1.3.2 Difference-in-Differences (DiD) Analysis of Attitude Shifts between TVHPI and CVHPI

This subsection presents a Difference-in-Differences graph (Figure 19) illustrating the shifts in attitude within each instructional group to visualize the differences in attitude change between the TVHPI and CVHPI.

The DiD approach relies on the parallel trends assumption, which underpins valid estimation of the treatment effect through the group × time interaction term. While a baseline difference existed in learners' attitudes, the DiD model leverages this interaction term to isolate the net change attributable to the intervention, making the baseline difference less critical as long as the relative change over time between groups is captured.

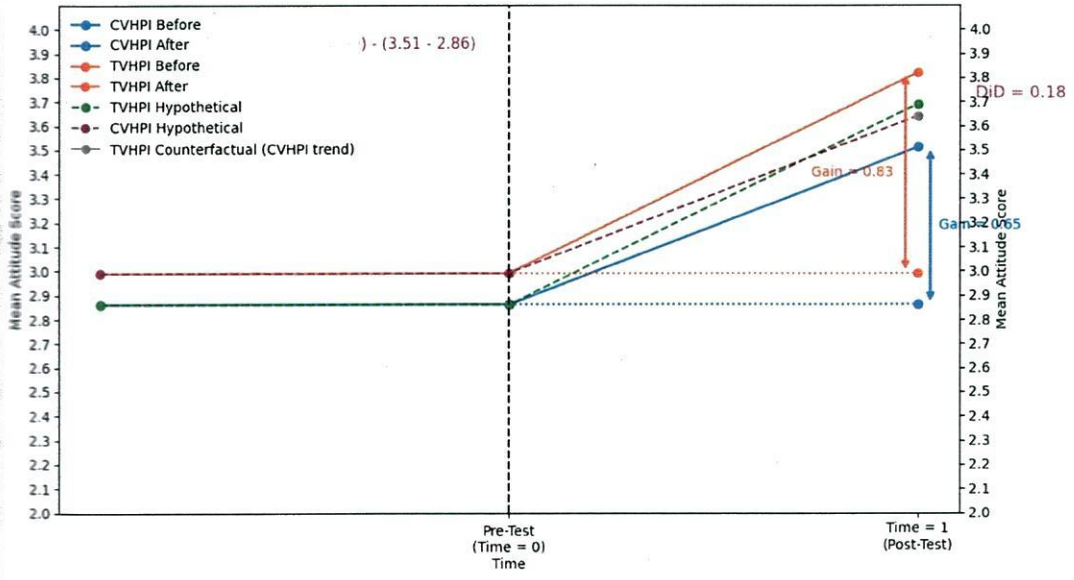


Figure 17: Difference-in-differences (DiD) analysis for learners' Attitudes

In Figure 17, the vertical dashed line is the intervention point that marks the start of the attitude change. In contrast, the solid blue line represents the trend for the CVHPI group, the solid red line represents the TVHPI group, and the purple and green dashed lines are hypothetical lines for TVHPI and CVHPI, respectively, to demonstrate how the trends of the two instructional strategies would appear if their starting point were interchanged. Lastly, the x-axis represents the period from pre-test (0) to post-test (1); moreover, the two dotted counterfactual horizontal lines represent the trend of the attitude (assumed to be constant) if the intervention was not implemented.

The trends for both the TVHPI and CVHPI show a forward upward movement from pre-test to post-test, signifying that both strategies led to some changes in the learners' mean attitudes across the intervention period. The TVHPI trend, representing the change in the mean attitude of learners who were taught using technology-enhanced Van Hiele Phased Instruction from pre-test to post-test, is above the CVHPI trend, implying that learners in the TVHPI group experienced more attitude change compared to their counterparts in the CVHPI group.

The hypothetical lines also indicate that these trends are not affected by the starting point, as interchanging the starting points between the two instructional groups shows that the counterfactual line for TVHPI would still be above the original CVHPI trend line. Furthermore, the hypothetical line for the CVHPI group remains below the original TVHPI trend line.

The gain in attitude for the TVHPI group is represented by the red arrow, and that for the CVHPI is represented by the blue arrow. As noted, the red arrow is longer than the blue arrow, implying that the TVHPI led to higher attitude gain than CVHPI. The DiD is the difference between the red line and the blue line.

In comparison, these results suggest that the TVHPI led to significantly greater improvement in learners' attitudes towards transformation geometry, further supporting the rejection of the second null

hypothesis (H02) and leading to the conclusion that TVHPI, as a technology-enhanced method, has a stronger and more lasting impact on learner engagement and learning outcomes in geometry. To conclude the test for this second hypothesis, a DiD-OLS model was fitted to isolate the net effect of the instructional strategies.

4.1.3.3 OLS Regression Analysis of Attitude Shifts by Instructional Strategy

To quantify and differentiate between the time and group effects on learners' attitude changes across the two instructional strategy groups, while accounting for baseline differences, a Difference-in-Differences model estimated via an ordinary least squares regression model was utilized, here referred to as the DiD-OLS model. The results are summarized in Table 11, followed by the equation form of the model in Table 12.

Table 11: DiD model Via Ordinary least squares (OLS) regression model

Effect	Coefficient	Std. Error	t-value	p-value
Intercept (Baseline CVHPI Pre-Test)	2.89	0.03	84.45	<0.001
Group Effect (TVHPI vs CVHPI at Pre-Test)	0.14	0.05	2.89	0.004
Time Effect (Post-Test vs Pre-Test for CVHPI)	0.59	0.05	12.19	<0.001
DiD Effect (TVHPI Post-Test Improvement Beyond CVHPI)	0.24	0.07	3.43	0.001

Table 12: Equation form of the DiD model Via OLS Model

$$Y_{it} = 2.89 + 0.14(Group_i) + 0.59(Time_t) + 0.24(Group_i.Time_t) + \epsilon_{it}$$

Where: Y_{it}: Overall Attitudes;

$Group_i$: 1 if the group is TVHPI, 0 if the Group is CVHPI;

$Time_t$: 1 if the time is post-test, 0 if the time is pre-test;

$Intercept$:	2.89 (baseline attitude for CVHPI at pre-test);
$\beta_1 = 0.14$:	The differences in pre-test attitudes between TVHPI and CVHPI.
$\beta_2 = 0.59$:	The change in CVHPI attitudes from pre-test to post-test, and
$\beta_3 = 0.24$:	The additional attitude improvement for TVHPI beyond CVHPI from pre-test to post-test.
ϵ_{it}	The error term, defined by $Y_{it}^{obderved} - Y_{it}^{Predicted}$ (the attitude that the model fails to explain.

Using the data from Table 11, the equation form of the DiD-OLS model is formulated in Table 12 to summarize the effect of the instructional strategies on learners' attitudes towards transformation geometry. The baseline attitude mean score of 2.89 for the CVHPI in the pre-test is significantly different from zero ($p < 0.001$) and with very precise variations ($Std.error = 0.03$). This means that the baseline value is reliable, and if the study is repeated with similar data, the baseline estimate of 2.89 would be closer to the true value most of the time.

Turning to the group effect that compares the TVHPI and CVHPI at pretest, it is observed that the coefficient is 0.14 with $p = 0.004$ and $Std.error = 0.05$, indicating significant initial differences between the learners' attitudes in the two instructional groups, with learners from the TVHPI group having a slightly higher attitude than those from the CVHPI group. The standard error is small, indicating that the estimated difference is reasonably precise.

Comparing the effect of time on attitude change within the CVHPI group, it is noted that the coefficient is 0.59, $p < 0.001$, revealing that learners from the CVHPI experienced attitude improvement even without technology after the intervention, with a small margin of uncertainty ($Std.error = 0.05$), indicating a very precise estimate.

Lastly, isolating the additional effect of the TVHPI in the post-test above and beyond what is reflected by the CVHPI group reveals a DiD effect coefficient of 0.25, $p = 0.001$, and $Std.error = 0.07$. This result suggests that learners from the TVHPI group experience an improvement in their attitudes,

0.24 points above their counterparts from the CVHPI group. This means that there is a benefit to using technology over conventional instruction. This benefit is statistically significant ($p = 0.001$) and reasonably precise, though not as tight as the other effect; this is expected since the coefficient is based on the difference in differences.

In a nutshell, the DiD-OLS model shows and supports the earlier results supporting that learners in both instructional strategies experience a shift in their attitudes towards transformation geometry from pre-test to post-test, and that the learners taught using Technology-enhanced Van Hiele Phased Instruction experienced higher attitude shifts compared to those taught using the conventional Van Hiele Phased Instruction. Confirming that the technology-based instruction is significantly superior to non-technology instruction strategies.

4.1.4 Discussion and Summary of Findings on Learners' Attitudes towards Transformation Geometry

4.1.4.1 Discussion

This first objective focused on establishing how learners' attitudes shifted from pre-test to post-test across the two instructional strategies within the Van Hiele Theory of Geometric Thought and the Constructivist Learning Theory framework. The results confirm that both instructional strategies significantly improved students' attitudes, with TVHPI demonstrating a more significant positive shift than CVHPI. These findings align with prior research on Van Hiele's instructional model and reinforce the theoretical argument that structured geometric instruction, especially when integrated with technology, enhances learner engagement, motivation, and conceptual understanding (Machisi & Feza, 2021; Uwurukundo et al., 2022). This further highlights the role of technology in supporting geometry learning and engagement among learners, aligning with the findings of Çavuş and Deniz (2022) and Hoyles and Noss (2009), who found that technology, though overlooked, significantly influences mathematical development.

However, Hoyles & Noss (2009) add that its influence depends on how the technology is integrated into the learning activities. This suggests that appropriate strategies for integrating technology into instruction must be considered to maximize technology's contribution to learning outcomes among learners.

The study's findings support the Theory of Reasoned Action, Ajzen & Fishbein (2005) and Colman (2015), which posits that attitudes directly influence behavioral intentions and learning outcomes. By showing a significant improvement in students' affective, behavioral, and cognitive attitudes, the results suggest that instructional strategies that foster positive attitudes toward mathematics may ultimately enhance engagement and achievement in geometry.

Moreover, the findings align with the constructivist perspective, emphasizing active learner engagement in constructing mathematical knowledge. The greater effectiveness of TVHPI, as evidenced by the Difference-in-Differences analysis, highlights the potential of technology to mediate and accelerate the learning process through dynamic visualization and interactive exploration. These findings also lay the groundwork for integrating technology in teaching and learning, which is one of the emphases of the new lower secondary curriculum in Uganda, thereby providing an approach that mitigates the technology-related implementation challenges reported by Kidega et al. (2023) that teachers in secondary schools face while implementing the CBC.

The study confirmed that Van Hiele Phased Instruction significantly improved learners' attitudes toward transformation geometry in conventional and technology-enhanced forms. The Wilcoxon Signed-Rank Test results demonstrated that learners in both groups exhibited positive attitude shifts from pre-test to post-test, providing empirical evidence that the Van Hiele Theory's structured approach not only guides conceptual progression but also attitude development (Alex & Mammen, 2016; Clements & Sarama, 2021). This finding is an extension of the Van Hiele theory, which was mainly developed to shape learners' level of geometric reasoning. However, it can also shape learners' attitudes. Moreover, the

effect is expanded by integrating technology such as GeoGebra into the phased instruction (Narh-kert & Sabtiwu, 2022; Soheila & Rosemaliza, 2018; Zengin, 2017).

The findings further align with prior research indicating that CVHPI effectively enhances geometric comprehension through structured exploration and teacher support (Abdullah & Zakaria, 2013a; Adeniji & Baker, 2022). However, the greater effectiveness of TVHPI in fostering positive attitudes suggests that technology enhances constructivist learning experiences, making abstract geometric concepts more engaging and accessible to students. This is consistent with Uwurukundo et al. (2021) and Zengin (2017), who found that GeoGebra improves learners' motivation and engagement in mathematics by providing dynamic representations of geometric transformations.

The study's results strongly support the hypothesis that Technology-Enhanced Van Hiele Phased Instruction (TVHPI) improves learners' attitudes toward transformation geometry more than Conventional Van Hiele Phased Instruction (CVHPI). The DiD analysis ($\beta_3 = 0.24$, $p = 0.001$) confirmed that the attitude improvement in TVHPI exceeded that of CVHPI, demonstrating that integrating technology into VHPI significantly enhances student engagement, curiosity, and confidence. This finding is consistent with Bora and Ahmed (2018) and Pavlovicova and Zahorska (2016), who found that instructional strategies integrating technology result in higher motivation and positive attitudes in mathematics.

The finding calls for mathematics teachers to integrate technology into their teaching to scale up learners' attitudes toward the subject, supporting engagement and self-directed learning, which, in turn, combined with other factors, would enhance the acquisition of both conceptual and procedural knowledge (Mendezabal & Tindowen, 2018).

The breakdown of attitude components further highlights the role of technology in enhancing different aspects of learner engagement. The affective component, which captures interest and confidence, showed the most significant improvement in the TVHPI group, suggesting that technology-integrated instruction makes learning more enjoyable and relatable to students (Clapper, 2014). The behavioral component, which reflects effort and engagement, also improved more significantly in TVHPI,

supporting research by Susilawati et al. (2017) that suggests interactive learning environments promote active participation and self-regulated learning. Finally, the cognitive component, which focuses on perceived relevance, demonstrated that TVHPI helped students better appreciate the real-world applications of transformation geometry, reinforcing arguments by Hwang and Son (2021) and Sinclair and Bruce (2015) that students develop a stronger understanding when they can visualize and manipulate geometric concepts dynamically. This implies that teachers of mathematics ought to use visualization tools to support learning in mathematics classrooms.

These findings further align with Constructivist Learning Theory, which posits that learners develop deeper conceptual understanding through active exploration and hands-on learning experiences (Allen, 2022). By allowing students to manipulate geometric figures in real time, TVHPI reduces cognitive load (Sweller, 2011) and helps students bridge the gap between abstract concepts and concrete understanding. This supports prior studies demonstrating that technology-assisted learning fosters deeper engagement and conceptual growth in mathematics (Adelabu et al., 2022; Celen, 2020; Dahal et al., 2022).

One of the study's significant contributions is its contextual relevance to Uganda's Competency-Based Curriculum (CBC). As the CBC emphasizes learner-centered instruction and conceptual understanding over memorization, the results suggest that TVHPI aligns more effectively with these curriculum goals than CVHPI. This is particularly relevant when teachers struggle to transition from traditional teacher-centered methods to more interactive pedagogies (Kidega et al., 2023).

From the researcher's perspective, the results affirm the hypothesis that instructional methods embedded with visualization, interaction, and student autonomy are more likely to yield positive affective responses toward geometry. The superiority of TVHPI over CVHPI also highlights the necessity of rethinking how geometry is taught in resource-constrained settings, encouraging low-cost technology integration to enhance learners' attitudes and engagement. In this regard, GeoGebra can be used as a display board in schools that can only afford a project, a laptop, or a computer. This approach assists

learners in visualizing the dynamic nature of the different mathematical variables and shapes compared to the use of a blackboard, whose use does not allow illustration of the dynamic nature of such variables and shapes.

In addition, the conventional Van Hiele phased instruction is not obsolete but can still be leveraged in areas that are technologically handicapped as a structured learner-centered instructional strategy. This is because, according to the current study and prior studies such as that of Abdullah and Zakaria (2013b), Alex and Mammen (2016), and Tahani (2016), the approach promises learner engagement, which is a function of their attitudes.

4.1.4.2 Summary

The first objective was to compare the effect of CVHPI and TVHPI on learners' attitudes toward transformation geometry. Findings show that both strategies significantly improved attitudes after the intervention, with TVHPI producing a greater positive shift than CVHPI. These results provide strong evidence against both null hypotheses, confirming that technology integration enhances learners' attitudes more effectively than the conventional approach.

4.2 Objective 2: Effect of Instructional Strategies on Learners' Van Hiele Level Progression

This section presents the results for the second objective, which examined the effect of instructional strategies, TVHPI and CVHPI, on learners' progression through Van Hiele's levels of geometric reasoning. The study tested two null hypotheses: First, that neither instructional strategy significantly improves learners' Van Hiele levels (H_{03}); nor second, that there is no significant difference in the degree of improvement between learners taught using TVHPI and those taught using CVHPI (H_{04}). The data used to address the hypothesis of this objective can be accessed via this link[8]. The findings are presented beginning with descriptive statistics, followed by reliability analysis and inferential statistics for hypothesis testing. The section concludes with a discussion of the findings and a brief summary.

4.2.1 Descriptive Statistics of Van Hiele Levels

Descriptive statistics, mainly mean, standard deviation, minimum and maximum scores, and frequency distributions, were computed to provide an initial overview of learners' progression through the Van Hiele levels for both TVHPI and CVHPI and to highlight general trends in learners' Van Hiele level scores from pre-test to post-test. Initially, the reliability of the Van Hiele achievement test was determined to find its psychometric soundness. The results yielded a Cronbach's alpha of 0.781, which falls within the acceptable range, indicating that the test items reliably measured learners' understanding of transformation geometry.

[8] https://tinyurl.com/ypae49vw

4.2.1.1 Frequency Distribution of Van Hiele Levels

This subsection outlines the frequency distribution of learners across the different Van Hiele levels at pre-test and post-test, disaggregated by instructional group. The distribution illustrates how learners were distributed across the different Van Hiele levels before and after instruction in both instructional groups, as shown in Table 13.

Table 13: Frequency distribution for Pre-VHL and Post-VHL Scores by Group

Van Hiele Levels	CVHPI		TVHPI	
	Pre-VHL	**Post_VHL**	**Pre-VHL**	**Post_VHL**
0. Pre-visualization	109 (44.5%)	22 (9.0%)	99 (41.6%)	12 (5.0%)
1. Visualization	75 (30.6%)	102 (41.6%)	80 (33.6%)	70 (29.4%)
2. Analysis	43 (17.6%)	82 (33.5%)	40 (16.3%)	92 (38.7%)
3. Abstraction	18 (7.4%)	37 (15.1)	19 (8.0%)	61 (25.6%)
4. Deduction	-	2 (0.8%)	-	3 (1.3%)

Table 13 presents the number of learners and percentage distribution in each Van Hiele level and instruction group in the pre-test and post-test. The Van Hiele levels include a pre-visualization level, representing students below the first formal Van Hiele levels at the pre-test. The visualization, analysis, abstraction, and deduction levels are the first four Van Hiele levels; the last (fifth) Van Hiele level was out of scope for this study, based on the study context.

For the CVHPI group, most of the learners (44.5%) were at the previsualization level before the intervention, and the number was reduced to 9.0% after the intervention. This indicates that the intervention probably facilitated the learners in the CVHPI to progress from pre-visualization to visualization. This led to an increase in the number of learners who reached the visualization level in the post-test (41.6%) compared to those at the same level (30.6%) before intervention. Moreover, few

learners in this group were at analysis (17.6%) and abstraction (7.4%) levels, while no learner in this group was at the deduction level at the inception of the intervention. However, after the interventions, there was a slight increase in the proportion of learners who achieved the analysis, abstraction, and deduction levels with proportions of 33.5%, 15.1%, and 0.8%, respectively.

Turning to the TVHPI, similar to the CVHPI group, most of the learners were at pre-visualizations (41.6%), which also reduced to 5.0% after the interventions, suggesting that the TVHPI strategy facilitated learners to move from pre-visualization to visualization. Expectedly, the proportion of learners in the visualization level reduced after the intervention. This suggests that the TVHPI facilitated learners to progress from the visualization level, and some learners could have progressed from pre-visualization directly to the analysis level. In this group, most of the learners achieved the analysis level (38.7%) after interventions.

Moreover, the proportion of learners who reached higher levels, abstraction, and deduction compared to the proportion at the pretest stage slightly increased, highlighting that some students achieved higher-order geometric reasoning following the instructional intervention. Overall, the learners progressed from lower to higher levels, with most progressing within pre-visualization, visualizations, and analysis, and very few progressing to abstraction and deductive levels.

4.2.1.2 Mean Van Hiele Levels by Instructional Group

The mean, maximum, and minimum values of the Van Hiele levels were calculated to provide a summarized overview of the learners' Van Hiele levels, which are presented in *Table 14* for each instructional strategy group.

Table 14: Descriptive Statistics for Van Hiele Scores by Instructional Group

Group	Pre-VHL Mean	Pre Min	Pre Max	Post-VHL Mean	Post Min	Post Max	Diff-VHL Mean
CVHPI	0.88	0.0	3.0	1.57	0.0	4.0	0.69
TVHPI	0.91	0.0	3.0	1.89	0.0	4.0	0.97

The results in Table 14 indicate that the means of both instructional groups were relatively low at the pre-test (0.88 for CVHPI and 0.91 for TVHPI). This suggests that learners began at lower Van Hiele levels, primarily at the pre-visualization or visualization levels. The minimum score at the pretest was zero, while the maximum score was 3, reinforcing the previous findings regarding many learners at the pre-visualization level, with no learners at the deduction level before the intervention for either group.

In the post-test, it is observed that the mean Van Hiele levels for both groups, TVHPI and CVHPI, increased (1.57 for CVHPI and 1.89 for TVHPI) compared to the pre-test mean, with TVHPI having a higher mean value than the CVHPI, indicating that most learners made at least one level of progression, with some making up to three-level improvements. These results suggest that both instructional strategies supported learners' progression through Van Hiele levels of geometric reasoning. The minimum and maximum Van Hiele level attained in the post-test are 0 and 4, respectively, for both instructional groups. This means some learners remained at the pre-visualization level after the intervention. Some learners have achieved up to the fourth level after intervention, suggesting that some learners could demonstrate deductive reasoning following instruction.

The mean difference in VHL is positive for both groups, with TVHPI showing a higher mean difference than the CVHPI. This implies that the instructional strategies positively impacted learners' Van Hiele levels of geometric reasoning. Also, the results suggest that the technology-based strategy demonstrated a higher effect in facilitating learners' progression through Van Hiele levels from pretest to post-test. Figure 18 visually represents the trend in the progression through Van Hiele levels from pre–test to post-test in both instructional strategies.

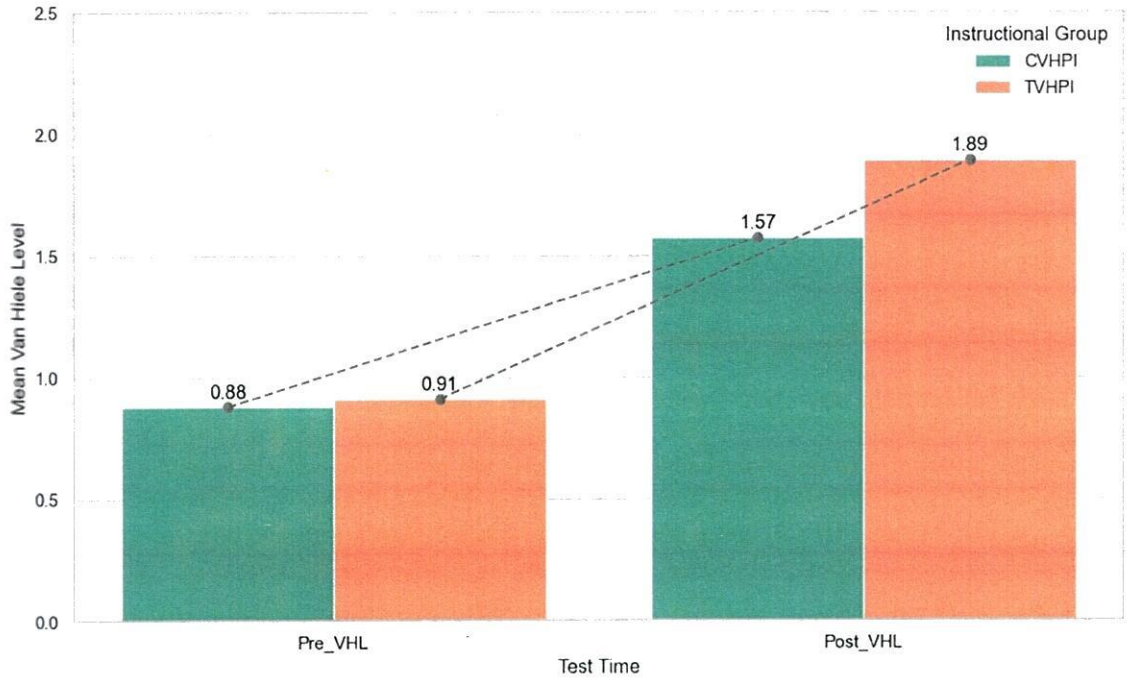


Figure 18: Mean Pre-Test to Post-Test Progression by Instructional Group

Figure 18 illustrates a graph showing a clear advancement in Van Hiele levels from the pre-test to the post-test for both instructional groups. The green bars represent the mean Van Hiele level for the CVHPI group, and the maroon color represent the mean Van Hiele level for the TVHPI group. As noted before, both groups have an upward slope line moving from left to right, with the CVHPI having a lower slope than the TVHPI, implying that the TVHPI had more impact on learners' advancement through Van Hiele levels.

4.2.2 Effect of Instructional Strategies on Van Hiele Level Progression

In this subsection, the third (H_{03}) and fourth (H_{04}) hypotheses are tested to draw necessary conclusions on the superiority of the instructional strategies. These examined whether the instructional strategies significantly improved learners' Van Hiele levels from pre-test to post-test, and whether there is a

significant difference in learners' progression between learners taught using TVHPI, who exhibited significantly greater progression in Van Hiele levels, and those taught using CVHPI.

4.2.2.1 Testing the Effectiveness of Instructional Strategies (H_{03}, H_{a_3})

The Wilcoxon Signed-Rank Test was used to determine whether each instructional strategy significantly improved learners' geometric reasoning. This non-parametric test was appropriate, given the ordinal nature of the Van Hiele levels and the previously established violation of normality. The test compared learners' pre-test and post-test Van Hiele scores within each group (CVHPI and TVHPI).

Although transformation geometry had not been formally taught to the learners before the pre-test, the Van Hiele model allows for assessing geometric reasoning independent of prior content knowledge. The model emphasizes learners' levels of reasoning, ranging from basic visualization to formal deduction, rather than focusing on specific topic mastery. Therefore, the pre-test served as a valid baseline for learners' geometric reasoning abilities. This enabled a meaningful comparison with post-test scores to determine whether the instructional interventions facilitated progression through the Van Hiele levels.

While it is expected that any structured instructional strategy may lead to some improvement, the purpose of the within-group comparison was to determine whether the instructional strategies resulted in statistically significant and theoretically meaningful gains in geometric reasoning as predicted by the Van Hiele model. These within-group improvements also provide a foundation for valid between-group comparisons by confirming that both interventions could produce change before assessing their relative effectiveness. Table 15 summarizes the results of the Wilcoxon signed test for within-group progression.

Table 15: Wilcoxon Signed-Rank Test Results for Within-Group Progression

Instructional Group	Test Statistic (W)	p-value	Interpretation
CVHPI	1164.5000	<.001	Significant change (reject H_{03})
TVHPI	133.0000	<.001	Significant change (reject H_{03})

The results in Table 15 show that both instructional groups experienced a statistically significant increase in Van Hiele levels from pre-test to post-test. In the CVHPI group, the Wilcoxon test produced a statistic of 1164.50 ($p < .001$), indicating a statistically significant difference between pre-test and post-test Van Hiele levels. Hence, the first part of the null hypothesis (H_{03}) was rejected, concluding that the learners in CVHPI experienced significant progression in geometry reasoning.

The TVHPI group had a statistic of 133.00 ($p < .001$), indicating significant progression within this group. Therefore, the second part of (H_{03}) is also rejected for the TVHPI group, suggesting that technology integration contributed to substantial development. Consequently, the instruction in each group positively influenced learners' advancement through the Van Hiele levels of geometric reasoning.

4.2.2.2 Comparison of Van Hiele Level Progression between Instructional Groups

To assess whether there was a statistically significant difference in learners' progression through Van Hiele levels across groups, a Generalized Ordered Logit model was fitted. The post-test Van Hiele level, treated as an ordered outcome, was regressed on the instructional group variable (0 = CVHPI, 1 = TVHPI). The model output, including the group effect and threshold (cut-point) parameters, is presented in Table 16.

Table 16: Generalized Ordered Logit (GOL) Model – Effect of Instructional Group on Post-Test Van Hiele Levels

Coefficient	Estimate (β)	Std. Error	z-value	p-value	Odds Ratio	Interpretation
Instructional Group (TVHPI)	0.661	0.169	3.91	<.001	1.94	Learners in the TVHPI group are 1.94× more likely to reach a higher Van Hiele level.
Threshold 0/1	-2.302	0.190	-12.10	<.001	0.10	Cut-point between Level 0 and Level 1
Threshold ½	0.843	0.075	11.24	<.001	2.32	Cut-point between Level 1 and Level 2
Threshold 2/3	0.496	0.068	7.34	<.001	1.64	Cut-point between Level 2 and Level 3
Threshold ¾	1.187	0.135	8.80	<.001	3.28	Cut-point between Level 3 and Level 4

From Table 16, the model reveals that learners' likelihood of achieving higher Van Hiele levels at the post-test can be significantly predicted. Explicitly, the coefficient for the TVHPI group was positive and statistically significant ($\beta = 0.661$, $p < .001$), and the corresponding odds ratio (OR = 1.94). The odds ratios not only indicate statistical significance but also provide insights into educational significance. For example, an odds ratio of 1.94 for learners in the TVHPI group implies that, after the intervention, students receiving technology-enhanced instruction were 1.94 times more likely to progress to a higher Van Hiele level than those in the CVHPI group.

In classroom terms, this translates into substantially increased chances of conceptual advancement. For instance, if 40% of students in a conventional class typically progress to the next level of geometric reasoning, an odds ratio of 1.94 suggests that approximately 70–77% of students in a technology-enhanced class could reach that level under similar conditions.

The threshold estimates (cut-points) represent the points on the latent scale that separate adjacent Van Hiele levels. Each is statistically significant, confirming that the model effectively distinguishes between levels. While these thresholds are not interpreted in the same way as predictors, they support the

internal logic of the model by structuring the progression from one level to the next. These results provide strong evidence that the technology-enhanced instructional strategy (TVHPI) was more effective in promoting learner progression across Van Hiele levels than the conventional strategy. This supports the rejection of H_{04}, which stated that there would be no significant difference between the two strategies. The next subsection presents the results estimating the net effect of the instructional strategy over time.

4.2.2.3 Difference-in-Differences (DiD) Analysis of Instructional Strategy Effect

Because the dependent variable (Van Hiele levels) is ordinal, a Difference-in-Differences framework was implemented using a Generalized Ordinal Logit model, an extension of ordinal logistic regression (hereafter referred to as DiD-GOL) to estimate the net effect of instructional strategy over time. The results are illustrated in Table 17

Table 17: DiD– GOL model of Van Hiele Levels

Predictor	Coefficient (β)	Std. Error	z-value	p-value	Odds Ratio	Interpretation
Group (TVHPI)	0.085	0.173	0.492	0.623	1.09	No significant difference in the pre-test
Time (Post)	1.416	0.172	8.255	<.001	4.12	Significant progression over time
Interaction	0.503	0.238	2.119	0.034	1.65	TVHPI led to a 1.65× greater gain in Van Hiele levels

Table 17 shows the results of the DID-GOL model with three predictors: the group, time, and interactions with the Van Hiele level output variable were treated as an ordered categorical variable. Learners in the TVHPI group had 1.09 times the odds of being at a higher Van Hiele level at pre-test compared to the CVHPI group, the group coefficient is 0.085 with p-value of 0.623, which is greater than 0.05, implying that the coefficient is not statistically significant and that there are no significant differences in the pre-test, as both groups stated at the same Van Hiele level.

For the time predictor, at post-test, learners had 4.12 times greater odds of being in a higher Van Hiele level than at pre-test, regardless of group. The coefficient is 1.416, with $p < 0.001$, and is statistically significant. This implies that the intervention (overall, across both groups) strongly affected learners' Van Hiele levels over time. These findings reveal that the learners in the technology-enhanced strategy group experience a significantly higher 65% chance of advancing to a higher Van Hiele level than their conventional variant counterparts.

Thus, the results strengthen the evidence to reject the null hypothesis (H_{04}), concluding that there was a significant difference in learners' Van Hiele level progression between the two groups and that the TVHPI provides additional benefits beyond the CVHPI. To visualize this difference, as DiD graph was fitted; the next subsection illustrates this graph.

4.2.2.4 Graphical Representation of Van Hiele Level Progression Using Difference-in-Differences Approach

To visually complement the statistical findings and predicted probabilities, a graphical illustration applying the Difference-in-Differences approach was used to show the change in mean Van Hiele levels from pre-test to post-test for both CVHPI and TVHPI, as illustrated in Figure 19.

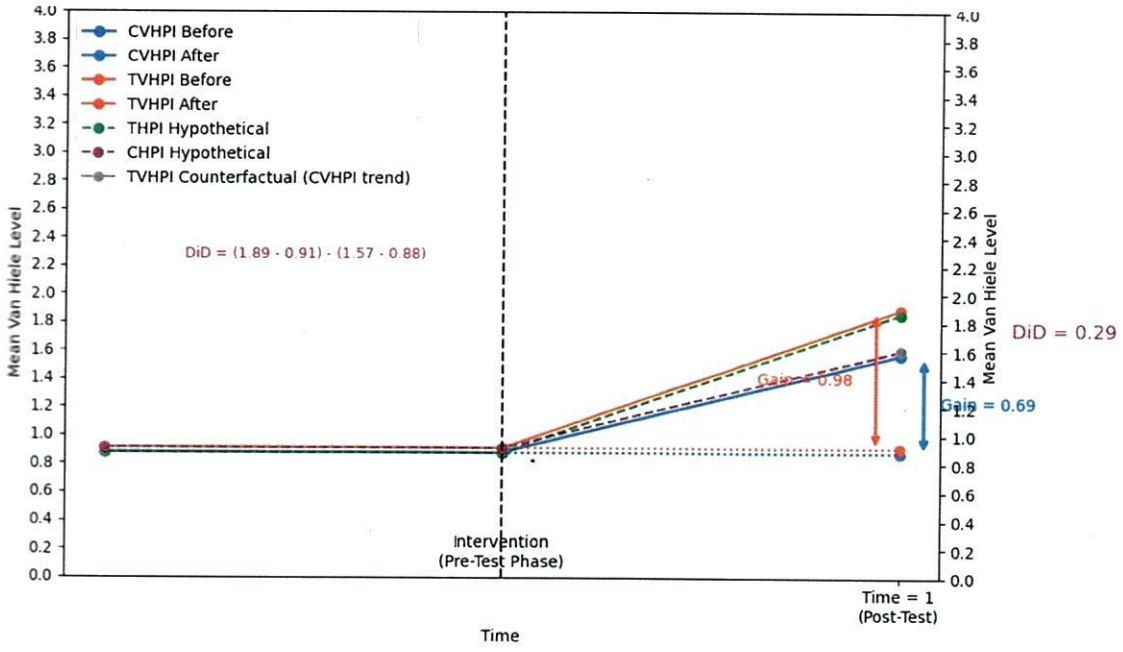


Figure 19: The Difference-in-Differences graph for Learners Van Hiele Levels

Figure 19 presents the DiD of the Van Hiele levels for the TVHPI (red solid line) and CVHPI (blue solid line). The green and purple dotted lines are the hypothetical lines for TVHPI and CVHPI if their pre-test levels were interchanged, respectively. In contrast, the horizontal dotted lines represent the trend if no intervention were performed. The vertical dotted line is the intervention point, indicating the start of the intervention effect on the Van Hiele levels.

The graph shows that the trend for both instructional groups is moving upwards, with TVHP increasing more than the CVHPI. Also, both groups had almost the same pretest mean, which minimized the initial level effect on strategies' effectiveness. The hypothetical lines reveal that if the starting points for the instructional groups were interchanged, TVHPI (green line) would still be above the CVHPI (purple line), reinforcing the limited effect of the initial Van Hiele levels on the effectiveness of the instruction.

The DiD is represented by the difference between the red and the blue arrows on the graph, which represent the differences in gains between the two instructional strategies. As shown, the gain by the TVHPI is greater than the gain by the CVHPI, highlighting that the TVHPI strategy facilitated a more conceptual understanding of transformation geometry than the CVHPI strategy.

4.2.2.5 Predicted Probabilities of Van Hiele Level Attainment

To further illustrate the impact of instructional strategies on learners' progression, predicted probabilities were computed to estimate the likelihood of attaining each Van Hiele level at the post-test, as illustrated in Figure 20.

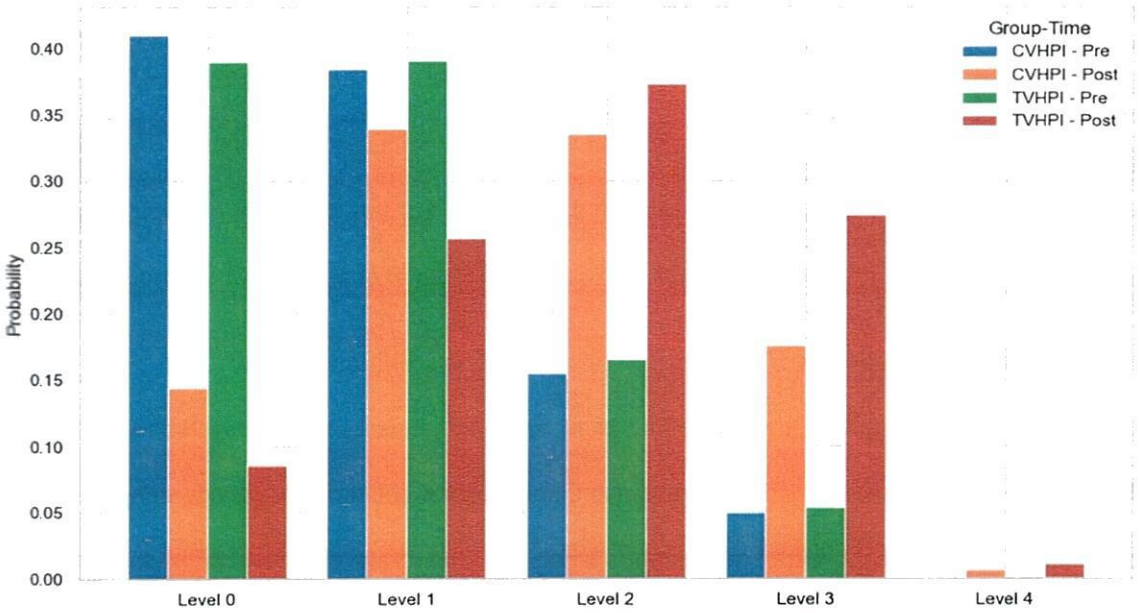


Figure 20: Predicted Probability of Achieving Each Van Hiele Level

The graph in Figure 20 represents the predicted probabilities for attaining different Van Hiele levels at pretest and posttest in each instructional group. Levels 0, 1, 2, 3, and 4 represent the pre-visualization, visualization, analysis, abstraction, and deduction, respectively.

For the learners in the CVHPI group, the probability of being at a particular level from level 0 to level 4 at the pretest decreases (Blue bars), implying that learners had more chances of being in lower levels than higher levels before the intervention. For the post-test, the probability of being in level 0 was less than in levels 1, 2, and 3 but higher than in level 4 (brown bars). This means that after intervention, learners had a higher chance of progressing from level 0 to other levels up to level 3. As noted, the probability of being in level 3 and level 4 was negligible compared to levels 1 and 2, meaning that learners had higher chances of attaining levels 1 and 2 than higher levels, 3 and 4, after the intervention.

For learners in the TVHPI, the probability of being at levels 0 and 1 was almost the same but higher than that for levels 2, 3, and 4 at the pre-test (green bars). Similar to the CVHPI, the probability of attainment decreased across levels with higher chances at lower levels compared to higher levels. At the post-test, the probability of remaining at level 0 was negligible compared to reaching levels 1, 2, and 3,

but it was higher than reaching level 4 (maroon bars). As noted, level 2 followed by level 3 indicated the highest probabilities compared to other levels, implying that learners had higher chances of achieving levels 2 and 3 compared to other levels. Level 4 had the lowest probabilities; however, its probability increased from zero in the pre-test to above zero in the post-test, signifying the role of the intervention in facilitating learners to progress to higher levels.

Generally, the probabilities of progression increased from pretest to post-test in all the instructional strategies (strengthening evidence to reject H_{03} and supporting H_{a3}), with CVHPI learners having a higher chance of achieving levels 1 and 2 compared to other levels, and TVHPI learners having higher chances of achieving levels 2 and 3 compared to other levels after the intervention. This means that the technology-based strategy provides learners with chances of attaining higher levels of Van Hiele compared to the non-technology variant, reinforcing the evidence to reject the null hypothesis (H_{04}) and leading to a conclusion that TVHPI facilitated learners' progression through Van Hiele levels compared to CVHPI.

4.2.3 Discussion and Summary of Findings on Learners' Van Hiele Levels

4.2.3.1 Dissection

The findings from the second objective indicate that both instructional methods, Technology-Enhanced Van Hiele Phased Instruction and Conventional Van Hiele Phased Instruction, effectively facilitated learners' progression through Van Hiele levels in transformation geometry, confirming that both methods significantly improve geometric reasoning from pre-test to post-test. This finding aligns with the foundational principles of the Van Hiele model, which posits that structured, phased instruction supports incremental cognitive development in geometry (Pujawan et al., 2020; Sert Celik & Kaleli Yilmaz, 2022; Vojkuvkova, 2012). The results further promise that the two approaches could be leveraged based on different contexts; for example, in areas where technology is limited or inaccessible, the conventional Van Hiele phased instruction can be used as a structured learner-centered strategy.

The success of both TVHPI and CVHPI in advancing learners' understanding reflects the inherent strengths of the Van Hiele phased instruction model, which organizes learning into structured phases starting from Visualization and progressing to Rigor. Each phase builds on the previous one, guiding students through increasingly complex reasoning tasks. The use of structured instruction aligns with studies that affirm the effectiveness of phased, level-based approaches in developing geometric reasoning (Abdullah & Zakaria, 2013a; DEMİR et al., 2023; Naufal et al., 2021; Senk et al., 2022; Tahani, 2016). The TVHPI is based on the VHPI that has been found to support learner progression, which has been reflected by the current study's findings, providing supporting evidence that VHPI is a practical approach.

This finding emphasizes the versatility and robustness of the Van Hiele model, independent of specific instructional tools. It resonates with cognitive-developmental theories, such as Bruner's constructivist approach, which emphasizes the importance of a structured learning sequence that

encourages students to construct knowledge at each stage actively emphasized by Xu et al. (2023), providing structured guidance through visualization, analysis, and, ultimately, abstraction (Scott & Palincsar, 2021). Since both methods significantly improve learners' geometric reasoning, educators can confidently adopt Van Hiele's phased instruction as a core approach to geometry education, regardless of the specific tools used. This approach aligns well with the Ugandan Competency-Based Curriculum, which emphasizes critical thinking and analytical skills. The results also suggest that while technology may enhance certain aspects of learning, the core Van Hiele phased instructional approach is highly effective in advancing students' geometric understanding. Consequently, for schools with limited access to technology, implementing Van Hiele phases through CVHPI still offers a robust and evidence-based approach to developing geometric reasoning.

Although both strategies facilitated progression, the analysis further demonstrated that Technology-Enhanced Van Hiele Phased Instruction is significantly more effective than Conventional Van Hiele Phased Instruction in advancing learners through Van Hiele levels of geometric understanding. This is possibly due to the dynamic nature of the TVHPI, where learners use GeoGebra to manipulate transformation shapes, enhancing their conceptual understanding. This finding stresses the added value of integrating technology into the Van Hiele phased instructional approach, which enhances the overall effectiveness of the model. This iś in support of existing studies, such as Dahal et al.(2019), KÜÇÜK and GÜN (2023), Narh and Sabtiwu (2022), Soheila and Rosemaliza (2018), Zengi (2017), and Zulnaidi and Zakaria (2012), that found similar findings about integrating technology into the Van Hiele phased instruction. The results further extend the Van Hiele theory, which was developed without a link to instructional media such as technology. Further, technology integration improves the effectiveness of van Hiele's phased instruction.

The success of TVHPI in promoting more excellent progression is consistent with research suggesting that technology supports more dynamic, interactive learning environments, which are particularly effective for advancing spatial and abstract reasoning (Clements & Sarama, 2021). It also

aligns with similar studies that investigated the use of GeoGebra in teaching different areas of mathematics, most of which found results in favor of GeoGebra in supporting learners' progression in VHL (T. Bekene, 2020; Dahal et al., 2022; Graeme Welch & Ponce Campuzano, 2023; Mudaly & Fletcher, 2019; Praveen & Leong, 2013; Soheila & Rosemaliza, 2018; Tamam & Dasari, 2021; Uwurukundo et al., 2022).

In this study, the technology component, particularly GeoGebra, allowed students to manipulate and visualize transformations in real time, reinforcing concepts that may remain abstract in conventional methods; this role of technology aligns with the findings of Viberg et al. (2023). Furthermore, Technology integration in TVHPI aligns with the Van Hiele model's emphasis on the gradual development of reasoning skills through structured phases (Vojkuvkova, 2012). Technology provides learners with immediate feedback at each level, from Visualization to Deduction, allowing them to directly observe the outcomes of transformations. This aligns with constructivist principles, which posit that learners build knowledge through active exploration (Allen, 2022). Several other studies indicate that technology-enhanced instruction provides "cognitive scaffolding," enabling students to visualize complex transformations and better understand abstract concepts (KÜÇÜK & GÜN, 2023; Vágová & Kmetová, 2019).

While this study's findings largely reinforce existing literature, some differences emerge in the effectiveness of technology-enhanced learning for all students. For instance, although Battista (2001) supports technology's role in advancing spatial reasoning, other studies, such as Hoyles and Noss (2009), caution that not all learners benefit equally from technology, particularly in under-resourced settings. This study's balanced group composition across rural and urban settings found no significant location-based difference, suggesting that technology's effectiveness can transcend geographical limitations, provided access is available. These findings emphasize the need for further research on contextual factors influencing technology integration, an area critical for CBC implementation in diverse Ugandan classrooms and beyond. The findings indicate that TVHPI aligns with CBC's learner-centered approach,

promoting active, exploratory learning that constructivist theory champions as essential for deep conceptual understanding.

In General, this objective addresses critical gaps in Uganda's secondary geometry education, particularly learners' challenges with progressing through the Van Hiele levels (VHL) of geometric understanding. By leveraging interactive technology, TVHPI enhances cognitive scaffolding through real-time visualization and feedback, providing a more engaging learning experience that promotes deeper understanding (Sinclair et al., 2012). Nonetheless, professional development in integrating technology with phased instruction is essential for optimal implementation, particularly in under-resourced areas, to maximize TVHPI's benefits and ensure sustainable improvements in diverse Ugandan educational settings (Hegedus et al., 2015). The effectiveness of phased, structured instruction, evident in this study, aligns with the Van Hiele model's cognitive framework and supports its integration within Uganda's Competency-Based Curriculum, which emphasizes learner-centered and problem-solving strategies.

4.2.3.1 Summary

The second objective examined the effect of instructional strategies on learners' progression through the Van Hiele levels of geometric reasoning. Both TVHPI and CVHPI significantly improved learners' Van Hiele levels from pretest to post-test, leading to the rejection of the null hypothesis that neither strategy would influence progression. However, TVHPI resulted in a greater advancement than CVHPI, with learners in the technology-enhanced group being more likely to reach higher levels of geometric reasoning. These findings confirm that while structured, phase-based instruction is effective in promoting conceptual development, integrating technology provides additional benefits by enhancing visualization, exploration, and engagement.

4.3 The Predictive and Mediating Roles of Learners' Attitudes in their Progression through Van Hiele Levels

Building on the first and second objectives, this section addresses the third objective of the study, which examines the predictive and mediating roles of geometry attitudes in learners' progression through Van Hiele levels across the two instructional strategies. Specifically, it investigates two key hypotheses: (H_{05}) assesses whether learners' attitudes significantly predict their Van Hiele levels before and after instruction, while (H_{06}) explores whether changes in attitudes mediate the relationship between instructional strategy and Van Hiele level progression.

GOL models were initially introduced to account for the ordinal nature of Van Hiele levels (outcome variable) and evaluate the direct and interactive effects of instructional strategy and attitude change. Bootstrapped mediation analysis was followed to formally estimate the indirect effect of attitude change on learners' progression, acknowledging that this analysis assumes an approximately continuous outcome for estimation purposes. As this objective builds directly on earlier analyses, descriptive statistics for attitudes and Van Hiele levels are omitted to avoid redundancy.

4.3.1 Predictive Role of Attitudes towards Geometry

To examine whether learners' attitudes significantly predicted their Van Hiele levels at both the pre-test and post-test stages, four GOL models were estimated. The first model used the pre-test overall attitude scores (TGAS1) to predict the initial Van Hiele levels (Pre-VHL), the second model used the post-test attitude scores (TGAS2) to predict the post-instruction Van Hiele levels (Post-VHL), while the third model used the pre-test overall attitude scores (TGAS1) to predict the post-Van Hiele level (Post-VHL). An additional model is estimated to assess whether learners' initial Van Hiele levels (Pre-VHL) significantly predicted their post-instruction levels (Post-VHL) while controlling for the instructional group.

4.3.1.1 Geometry Attitudes as Predictors of Pre-Test and Post-Test Van Hiele Levels

Three ordered logit models were estimated to test the relationship between learners' attitudes toward geometry and their Van Hiele levels, as outlined in Hypothesis H_{05}. Each model included the instructional group variable (TVHPI vs. CVHPI) and either pre-test attitudes (TGAS1), post-test attitudes (TGAS2), or the cross-predictive effect of TGAS1 on post-instruction Van Hiele levels. The results of the GOL models for each case are summarized in Table 18.

Table 18: Comparison of Ordered Logit Models (Attitudes and Van Hiele Levels)

Predictor / Threshold	**Model 1 (TGAS1 → Pre_VHL)**	**Model 2 (TGAS2 → Post_VHL)**	**Model 3 (TGAS1 → Post_VHL)**
Attitude Score	0.0155 (p = 0.911)	0.0889 (p = 0.631)	0.0667 (p = 0.621)
Instruction-Group	0.0753 (p = 0.656)	0.6274 (p = 0.001)**	0.6497 (p <.001)**
Threshold: 0/1	–0.1956 (p = 0.642)	–1.9916 (p = 0.003)**	–2.1087 (p < .001)**
Threshold: 1/2	0.3269 (p <.001)**	0.8435 (p <.001)**	0.8430 (p < .001)**
Threshold: 2/3	0.3238 (p = 0.003)**	0.4956 (p < .001)**	0.4958 (p < .001)**
Threshold: 3/4	–	1.1864 (p <.001)**	1.1866 (p < .001)**

As shown in Table 18, the attitude variable was not a statistically significant predictor in all three models. Specifically, TGAS1 did not significantly predict either Pre_VHL (Model 1) or Post_VHL (Model 3), and TGAS2 also failed to significantly predict Post_VHL (Model 2). These findings suggest that learners' attitudes, whether measured before or after instruction, did not directly influence their placement along the Van Hiele levels of geometric reasoning.

In contrast, the instructional group variable was a significant predictor in both post-test models (Models 2 and 3), with coefficients of 0.6274 ($p = 0.001$) and 0.6497 ($p < 0.001$), respectively. This result indicates that learners exposed to technology-enhanced Van Hiele instruction (TVHPI) were significantly more likely to attain higher Van Hiele levels than those in the conventional group (CVHPI), regardless of their initial attitudes. This reinforces the positive effect of the instructional strategy on learners' conceptual development.

In Model 1, the threshold between level 0 and level 1 (–0.1956, $p = 0.642$) was not statistically significant, indicating that learners at the lowest Van Hiele levels could not be reliably distinguished before instruction. However, the thresholds for transitions from level 1 to 2 ($p < .001$) and from level 2 to 3 ($p = .003$) were statistically significant, suggesting that while most learners had limited and comparable geometric understanding at baseline, a smaller group had already begun to show distinct conceptual progress.

In Models 2 and 3, all thresholds were statistically significant, confirming that the Van Hiele levels were meaningfully distinct at the post-test stage. In contrast, Model 1 showed only partial significance among the thresholds, with the 1/2 and 2/3 transitions significant, while the 0/1 threshold was not. This pattern supports the notion that Van Hiele's levels became more clearly separated after instruction.

The results generally imply that neither initial nor final attitudes had a meaningful statistical effect on learners' placement along the Van Hiele geometric reasoning continuum. As such, learners' cognitive advancement could not be explained by their affective, behavioral, or cognitive orientations toward geometry. Therefore, there was no significant statistical evidence to reject the null hypothesis (H_{os}), indicating that geometry attitudes did not significantly predict learners' progression through Van Hiele levels.

4.3.1.2 Predictive Role of Pre-Instruction Van Hiele Level on Post-Instruction Outcomes

To further understand learners' progression through Van Hiele levels, an additional model was estimated to assess whether learners' initial Van Hiele levels (Pre-VHL) significantly predicted their post-instruction levels (Post-VHL) while controlling for the instructional group. This analysis complements the earlier attitude-based models by establishing whether conceptual readiness at baseline played a role in learners' geometric development. The results are presented in Table 19.

Table 19: Ordered Logit Model Predicting Post-Test Van Hiele Level from Pre-VHL and Group

Variable	Coefficient	Std. Error	z-value	p-value	95% CI
Pre_VHL	1.7417	0.128	13.605	$p < .001$	[1.491, 1.993]
Instruction-Group	0.8663	0.183	4.732	$p < .001$	[0.507, 1.225]
Threshold: 0/1	–1.4252	0.201	–7.095	$p < .001$	[–1.819, –1.032]
Threshold: 1/2	1.0269	0.071	14.496	$p < .001$	[0.888, 1.166]
Threshold: 2/3	0.9370	0.072	13.002	$p < .001$	[0.796, 1.078]
Threshold: 3/4	1.4310	0.115	12.392	$p < .001$	[1.205, 1.657]

The model in Table 19 reveals that Pre-VHL was a strong and statistically significant predictor of Post-VHL ($\beta = 1.7417$, $p < .001$). This finding indicates that learners who began with higher levels of geometric reasoning were much more likely to advance further by the end of the intervention. In other words, prior conceptual understanding strongly influences future learning gains. This has important implications: while the instructional strategy plays a role, learners' readiness at the start of instruction remains critical in determining the depth of conceptual progression. It suggests that some learners may benefit more from differentiated or scaffolded instruction to ensure equitable progression, especially for those starting at lower Van Hiele levels.

The Instruction-Group variable also remained statistically significant ($\beta = 0.8663$, $p < .001$), confirming that the Technology-Enhanced Van Hiele Phased Instruction (TVHPI) approach continued to exert a positive influence on learners' outcomes, even after accounting for their initial Van Hiele levels. This strengthens the evidence that TVHPI was beneficial to already high-achieving learners and had an independent effect on promoting growth in geometric reasoning.

All threshold parameters were statistically significant ($p < .001$), indicating that the model reliably distinguished among the ordered categories of the post-VHL variable. This supports the appropriateness of the ordered logit model and confirms that the progression through Van Hiele levels was meaningful and statistically valid.

4.3.2 Mediating Role of Attitude Change in Instructional Effects

This subsection addresses the second hypothesis under the third objective, specifically testing whether changes in learners' geometry attitudes (ΔTGAS) mediate the effect of instructional strategies on learners' progression through Van Hiele levels. The analysis builds on the earlier finding that the instructional strategy significantly predicts Van Hiele's progression and attitude shifts.

Generalized Ordered Logit models were estimated to assess mediation. The first model examined the direct effect of instructional strategy (TVHPI or CVHPI) on Van Hiele progression without including attitude change. The second model introduced attitude change (ΔTGAS) to determine whether its inclusion reduced the direct instructional effect, a key criterion for establishing mediation. A third model was tested for moderated mediation by including an interaction term between ΔTGAS and the instructional group to explore whether the mediating role of attitude change differs between the two instructional conditions. Since GOL does not formally estimate indirect effects, a complementary bootstrapped mediation analysis was conducted using 5000 resamples. Table 20 summarizes the results of the GOL models, while Table 21 summarizes the results of the bootstrapped mediation.

Table 20: Ordered Logit Results – Instructional Strategy, Attitude Change, and Moderation

Variable	Model 1: Direct Effect	Model 2: Mediation (ΔTGAS)	Model 3: Moderated Mediation (ΔTGAS × Group)
Group	0.6480 (p = 0.000)***	0.6741 (p = 0.000)***	1.1227 (p < .001)***
Att-Shift (ΔTGAS)	–	–0.0997 (p = 0.565), ns	0.1843 (p = 0.434), ns
ΔTGAS × Group	–	–	–0.6210 (p = 0.075), †
Threshold 0/1	0.8911 (p < .001)***	0.8913 (p < .001)***	0.8921 (p < .001)***
Threshold 1/2	0.8791 (p < .001)***	0.8798 (p < .001)***	0.8858 (p < .001)***
Threshold 2/3	1.0355 (p < .001)***	1.0359 (p < .001)***	1.0387 (p < .001)***

**** $p < 0.001$, † $p < 0.10$, ns = not significant.*
Dependent Variable: Diff_VHL (Van Hiele Level Progression)

Table 20 presents three models with results on the direct effect, mediation, and moderated mediation. Model 1 examined whether the type of instructional strategy directly influenced learners' progression in Van Hiele levels. The coefficient for the Group variable was statistically significant (β = 0.6480, $p < .001$), indicating that learners in the TVHPI group were significantly more likely to progress in their geometric reasoning levels than their CVHPI counterparts. This highlights the effectiveness of incorporating technology such as GeoGebra in facilitating a deeper conceptual understanding of transformation geometry. The threshold estimates in this model (e.g., transitions from level 0 to 1, 1 to 2, etc.) were all statistically significant, confirming the reliability of level distinctions and suggesting meaningful transitions in learners' conceptual development during the intervention.

In Model 2, attitude change (ΔTGAS) was added to examine whether it mediated the effect of instructional strategy on learners' progression. The coefficient for ΔTGAS was not statistically significant

(β = –0.0997, p = 0.565), suggesting that changes in learners' attitudes toward geometry did not significantly influence their movement across Van Hiele levels. This implies that although some learners may have improved their attitudes during the intervention, these changes were not directly responsible for their conceptual gains. The Group variable remained statistically significant (β = 0.6741, $p < .001$) even after including attitude change. This reinforces the finding that the instructional approach, particularly the use of technology in TVHPI, had a strong and independent effect on learner progression. In essence, attitude change did not explain (mediate) the instructional effect, so the mediation hypothesis was not supported.

Model 3 tested whether the mediating effect of attitude change varied depending on the instructional strategy by including an interaction term (ΔTGAS × Group). The interaction term was marginally non-significant (β = –0.6210, p = 0.075), suggesting a possible trend that the role of attitude change may differ between TVHPI and CVHPI groups. Specifically, the negative coefficient implies that attitude change might have a slightly more substantial effect in the CVHPI group (which lacks technological support), where motivation and personal drive are more essential for learning. On the other hand, in the TVHPI group, learners might progress regardless of their attitudinal shift, potentially due to the visual and interactive affordances of GeoGebra that support conceptual understanding directly. The Group variable in this model showed an even more substantial effect (β = 1.1227, $p < .001$), confirming that the instructional method remains a powerful determinant of learner progression, even when potential moderating effects of attitude are accounted for.

The GOL models provide partial evidence regarding the absence of mediation by showing that the mediator (attitude shift) is not a significant predictor, and there is no interaction effect. However, a bootstrapped mediation analysis was necessary to determine whether a statistically significant indirect effect exists conclusively.

To formally test the mediating role of attitude change (ΔTGAS) in the relationship between instructional strategy and Van Hiele level progression (Diff_VHL), a bootstrapped mediation analysis was performed using 5000 resamples, as illustrated in Table 21.

Table 21: Bootstrapped Mediation Analysis of the Effect of Instructional Strategy on Van Hiele Level Progression via Attitude Change (ΔTGAS)

Path	**Coefficient**	**p-value**	**95% CI**
a (Group → Att-Shift)	0.237	$p < .001$	[0.148, 0.325]
b (Att-Shift → Diff_VHL)	0.033	$p = .642$	[–0.105, 0.171]
Indirect (a × b)	–0.008	$p = .608$	[–0.043, 0.024]
Direct (Group → Diff_VHL controlling for Att-Shift)	0.289	$p < .001$	[0.147, 0.431]

The results in Table 21 indicated that instructional strategy had a significant positive effect on attitude change ($\beta = 0.237$, $p < .001$) and a substantial total effect on VHL progression ($\beta = 0.281$, $p < .001$). However, the path from attitude change to VHL progression was not significant ($\beta = 0.033$, $p = .642$), and the indirect effect of instructional strategy through attitude change was also not significant ($\beta = –0.008$, $p = .608$, 95% CI [–0.043, 0.024]). The direct effect of instructional strategy on VHL progression remained significant even after controlling for attitude change ($\beta = 0.289$, $p < .001$). These results suggest that attitude change did not mediate the effect of instructional strategy on learners' progression through Van Hiele levels, thereby supporting the decision to retain the null hypothesis (H_{06}).

4.3.3 Discussion and Summary of Findings on the predictive and mediation role of Attitudes

4.3.3.1 Discussion

The third objective of the study investigated the predictive and mediating role of learners' attitudes in the relationship between instructional strategies and learners' Progression through Van Hiele levels. Further, it explored whether the mediation pathways differed significantly between CVHPI and TVHPI. The results revealed that neither pre-test attitudes (TGAS1), post-test attitudes (TGAS2), nor change in attitudes (ΔTGAS) significantly predicted learners' Van Hiele levels, as shown by the non-significant coefficients across all three GOL models.

Additionally, introducing ΔTGAS in the mediation models did not reduce the effect of instructional strategy on Van Hiele progression, nor did the interaction term (ΔTGAS × Group) achieve statistical significance. This finding aligns with prior research suggesting that while attitudes play a critical role in fostering motivation and engagement, their direct impact on short-term learners' Progression through Van Hiele levels may be limited in structured instructional contexts like Van Hiele-based interventions (Hwang & Son, 2021; Yea-Ling & Wei-Shin, 2022). Similarly, a study by Mogari (2003) found no statistically significant relationship between attitudes and achievement in Euclidean geometry, highlighting that attitudes are not the only predictor of learners' conceptual understanding.

Educational theories such as the Van Hiele model emphasize cognitive progression as a primary driver of learning outcomes. The minimal role of attitudes as mediators suggests that attitudes may act as secondary enhancers in environments emphasizing conceptual learning rather than primary pathways to achievement. This finding challenges assumptions in some motivational theories, such as self-determination theory, which posits that internal attitudes are central to performance outcomes (Ryan & Deci, 2017).

The results, however, show that the interaction of instruction directly influences learners' progression. These significant direct effects observed for both CVHPI and TVHPI on learners' Progression highlight the strength of these instructional strategies in enhancing learners' understanding of transformation geometry. The structured, phase-based progression emphasized by the Van Hiele model appears to effectively scaffold learners' cognitive development. TVHPI, which integrates dynamic, technology-enhanced tools like GeoGebra, demonstrated a more substantial effect than VHPI. This finding is consistent with constructivist theories and dual coding theory, which highlight the role of visualization and interaction in enhancing conceptual understanding (Chisenga & Alfred, 2021; Mthethwa et al., 2020b; Nzaramyimana et al., 2021; Zulnaidi & Zakaria, 2012). TVHPI's ability to support learner progression compared to CVHPI aligns with prior studies that highlight the potential of technology to transform mathematics education (Clements & Sarama, 2021).

The interactive features of GeoGebra likely enabled learners to visualize transformations more effectively, enhancing comprehension and retention as has been similarly reported in prior studies (Adelabu et al., 2022; Bekene & Machaba, 2022; Mthethwa et al., 2020a; Mudaly & Fletcher, 2019; Vágová & Kmetová, 2019). In addition, the combination of structured phases and interactive visualization tools appears to enhance learners' conceptual understanding directly, bypassing the need for attitudinal mediation (Ngwabe & Felix, 2020; Zulnaidi et al., 2020). Highlight the value of integrating technology into mathematics instruction. Dynamic tools like GeoGebra should be prioritized in curricula to enhance visualization and engagement.

The moderated mediation analysis revealed no significant differences in the mediation pathways between CVHPI and TVHPI. While TVHPI exhibited more substantial direct effects on Van Hiele level progression, the lack of significant differences in indirect effects suggests that the mechanisms driving attitudinal changes are largely similar across both instructional strategies. However, a marginal interaction effect was observed, indicating that attitude change may exert a slightly greater influence on progression in conventional instruction than in technology-enhanced settings. This nuance suggests that while

attitudes do not play a strong mediating role overall, they could have context-dependent relevance, particularly in traditional approaches. These findings underscore the robustness of the Van Hiele framework, which provides a common cognitive structure for both conventional and technology-integrated methods (Naufal et al., 2021; Robert, 2018).

The findings challenge the centrality of attitudinal changes in models of mathematics achievement, particularly in structured, outcome-driven contexts. While motivational theories such as expectancy-value theory emphasize the role of attitudes, the current results suggest that cognitive factors overshadow their immediate impact on performance, as, as emphasized in the Van Hiele model (Bandura, 1994).

These findings contradict several earlier studies that reported a positive relationship between attitudes and mathematical achievement. For example, Karjanto (2017) confirmed a positive correlation between mathematics achievement and attitudes. However, in the present study, while attitudes significantly improved following instruction (especially under TVHPI), they did not translate into measurable gains in geometric reasoning, as reflected by Van Hiele level changes.

This divergence may be attributed to the unique cognitive demands of the Van Hiele model, which posits that learners must progress through qualitatively distinct levels of reasoning (Van Hiele, 1986). Each level requires specific forms of instruction to enable transition, and mere positive disposition may not suffice to facilitate this movement.

According to Moru et al. (2021), progression through Van Hiele levels necessitates guided discovery and structured activities that align with learners' cognitive readiness. Hence, affective improvements, though beneficial for engagement, may not be substantial enough to trigger level advancement independently without instructional scaffolding.

The significant predictive power of Pre-VHL on Post-VHL ($p < .001$) reinforces the Van Hiele theory's emphasis on conceptual prerequisites. Learners with a stronger initial foundation in geometric

reasoning are more likely to progress, consistent with constructivism theory of learning that emphasizes that learning is through interaction with the environment and experiences (Mcleod, 2023)

The enduring significance of instructional strategy (particularly TVHPI) in all models further suggests that effective teaching methods, especially those leveraging technology, directly influence conceptual learning. This aligns with findings of Mthethwa et al. (2020b), and Tamam and Dasari (2021), who highlight the potential of dynamic geometry environments like GeoGebra in fostering deeper spatial reasoning and geometric understanding. The visual and interactive affordances of technology can act as cognitive amplifiers, supporting learners in grasping abstract geometric relationships.

From the researcher's perspective, these findings offer several implications. First, they suggest that while attitudes matter, they are not stand-alone predictors of conceptual understanding in transformation geometry. Instructional strategy and initial cognitive readiness remain more robust determinants of progression. Second, technology-enhanced instruction (TVHPI) appears to provide cognitive support that can, to some extent, compensate for variability in learner attitudes. This has practical significance for classroom implementation, particularly in resource-limited contexts where conventional instruction remains dominant. Efforts to foster positive attitudes should focus on long-term engagement and motivation rather than short-term performance gains. Strategies such as collaborative learning, real-world applications, and self-reflective practices may complement instructional frameworks.

4.3.3.1 Summary

This third objective examined whether learners' attitudes toward geometry predicted their progression through the Van Hiele levels and whether attitude change mediated the effect of instructional strategy. The results showed that neither pre-instruction nor post-instruction attitudes significantly predicted progression, and attitude change did not mediate the instructional effect, leading to the non-rejection of both null hypotheses (H_{05} and H_{06}). However, instructional strategy, particularly TVHPI, had

a strong positive effect on progression, and pre-VHL emerged as a significant predictor of post-VHL, underscoring the role of prior understanding. A marginal interaction effect indicated that attitude change may have a limited moderating role in conventional instruction, although this effect was not strong enough to alter the main conclusions. These findings are summarized in Figure 21, which visualizes the tested predictive and mediating relationships.

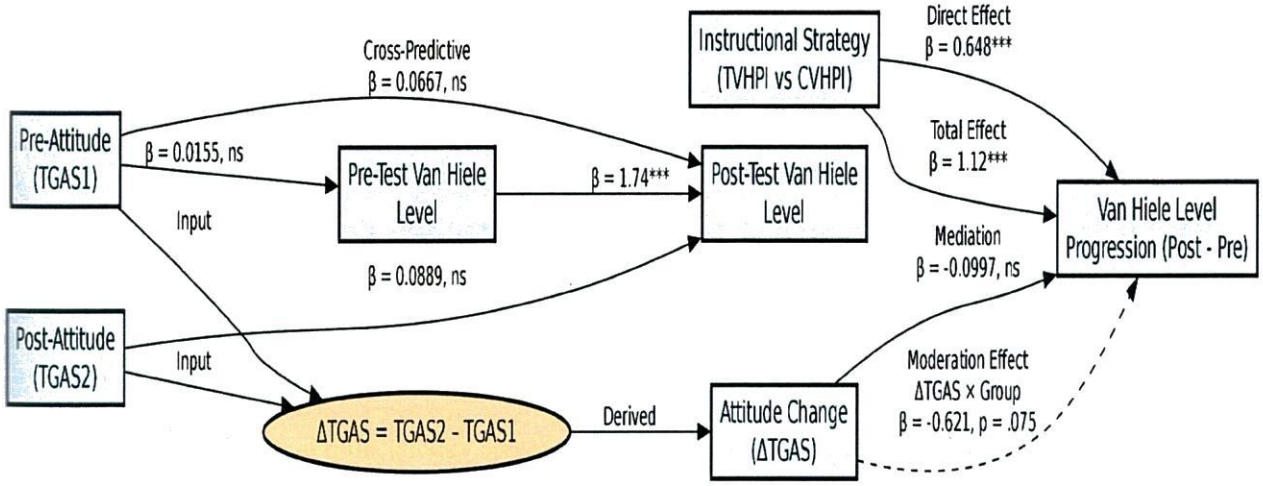


Figure 21: Predictive and Mediating Relationships between Attitude and Van Hiele Levels.

4.4. Learners' Experiences and Understanding of Transformation Geometry

This section addresses the fourth objective, which is to examine how learners' experiences in terms of challenges, support needs, and perceptions of the effectiveness of CVHPI and TVHPI in learning transformation geometry differ. Guided by two research questions, the analysis draws on qualitative data from post-intervention learner interviews, and one manuscript was drawn from this objective[9].

The study sample comprised 48 learners, with equal representation across gender, school location, achievement level, and instructional strategy. These demographic factors were included to create a balanced, representative interview sample of the larger study population, but they were not intended for comparative analysis. The main analytical focus remained on differences arising from the instructional approaches themselves.

To protect participant privacy, all data were anonymized before analysis. Each learner was assigned a unique code that reflected key contextual characteristics, such as gender, location, achievement level, and instructional strategy, without using real names or identification numbers. This coding preserved confidentiality while retaining essential contextual detail for meaningful interpretation.

The data underwent thematic analysis in Excel, applying built-in functions to organize and code responses systematically. From this process, two major themes emerged, capturing learners' experiences with CVHPI and TVHPI in studying transformation geometry. The full qualitative dataset is available via the provided access link[10].

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[9] Ndungo, I., Balimuttajjo, S., & Akugizibwe, E. (2025)._Towards Improved Geometry Instruction: Learners' Experiences with Conventional and Technology-Enhanced Van Hiele Phased Instruction. *The Electronic Journal for Research in Science & Mathematics Education.* 29(2), 33-56. https://ejrsme.icrsme.com/article/view/23815

[10] https://tinyurl.com/2de4aa2r

4.4.1 Key Themes from Learners' Reflections and Experience

The themes emerged from a detailed process of coding and thematic analysis of the interview transcripts, which revealed deep insights into learners' perceptions, challenges, and the instructional elements shaping their understanding of transformation geometry. The narratives from learners' reflections and experiences converged into two main themes: Challenges and Support Needs, and Instructional Effectiveness and Learner Satisfaction.

Challenges and Support Needs

Learners' voices painted a clear picture of the barriers they encountered in learning geometry and the kinds of support they believed could help them progress. This theme draws from three interconnected categories: Learning Challenges and Obstacles, Metacognition, and Self-Regulated

In their accounts, many learners described struggling to follow step-by-step procedures, making sense of abstract concepts, and gaining access to essential resources like geometric tools or reference materials. These procedural and conceptual difficulties often slowed their progress and diminished confidence. Alongside this, their narratives revealed how self-regulation, such as the ability to stay focused, reflect on their understanding, and remain motivated, was greatly influenced by the teaching approach. Where methods encouraged active participation and reflection, learners reported feeling more in control of their learning.

They also spoke of the value of external assistance, be it through teacher guidance, structured practice opportunities, or tailored interventions, which they saw as vital in overcoming their difficulties. Together, these stories highlight that successfully tackling learning challenges in transformation geometry requires a combination of internal strategies and external supports.

Instructional Effectiveness and Learner Satisfaction

The second theme illuminated the ways in which teaching methods shaped learning outcomes and how well they aligned with students' expectations. This theme arose from five categories: Instructional Support and Guidance, Learning and Differentiated Instruction, Practical Learning and Reinforcement through Practice, Role of Instructional Strategy in Learning, and Learners' Expectations.

Learners consistently described how clear explanations and ongoing guidance from teachers helped them resolve misunderstandings and strengthened their confidence. Those who experienced instruction tailored to their abilities and learning styles felt more included and better able to keep pace with the content. They particularly valued opportunities for practical engagement, applying concepts in hands-on tasks and reinforcing learning through repeated practice, which they associated with deeper understanding. The role of the instructional strategy itself was also significant; approaches that incorporated visual aids, step-by-step demonstrations, or interactive tools often sparked curiosity and engagement. Finally, the alignment between teaching methods and learners' expectations emerged as a key factor in fostering both satisfaction and motivation. When instruction met or exceeded what they anticipated, learners were more inclined to participate actively and persist through challenges.

In essence, these narratives reveal a complex interplay between the quality of instructional delivery, the strategies employed, and learners' perceptions. Together, the two themes highlight that improving geometry learning is not solely a matter of content delivery, but of harmonizing effective strategies, learner-centered approaches, and targeted support to create meaningful and engaging learning experiences.

4.4.2 The Challenges and Support Needs Associated With TVHPI & CVHPI

This subsection focuses on the challenges and support needs that emerged about both Conventional Van Hiele Phased Instruction (CVHPI) and Technology-Enhanced Van Hiele Phased Instruction (TVHPI). It

highlights the specific difficulties learners faced in grasping geometric concepts, as well as the types of assistance required to strengthen their understanding, particularly through practical, hands-on activities and personalized guidance.

The analysis highlights how learners navigated procedural steps, interpreted abstract concepts, and utilized available learning tools, identifying the points where targeted intervention was needed. Hands-on training was seen as vital for translating theoretical understanding into practical skills, while individualized support helped address unique learning gaps and foster confidence. Figure 22 presents a visual map showing how this theme, Challenges and Support Needs, developed from underlying categories and codes during the thematic analysis.

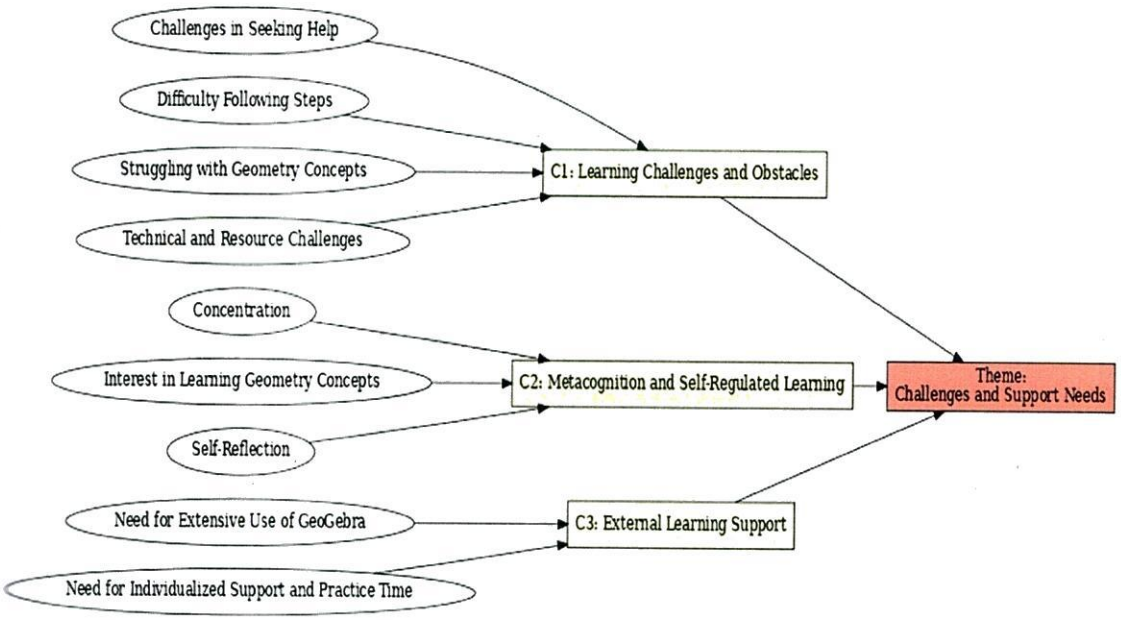


Figure 22: The emergence of Theme 1 from the categories and codes

Table 22 complements this by quantifying the frequency of each code within its category, providing a clear view of how often specific challenges or support needs appeared in the dataset for Theme 1.

Table 22: A count of codes and categories for Theme1

Row Labels	TVHPI	CVHPI	Total
Challenges and Support Needs	**77**	**89**	**166**
Learning Challenges and Obstacles	**49**	**60**	**109**
Challenges in Seeking Help	1	0	1
Difficulty Following Steps	0	3	3
Struggling with Geometry Concepts	16	38	54
Technical and resource challenges	32	19	51
Metacognition and Self-Regulated Learning	**2**	**9**	**11**
Concentration	2	0	2
Interest in learning geometry concepts	0	2	2
Self-Reflection	0	7	7
External Learning Support	**26**	**20**	**46**
Need for Extensive Use of GeoGebra for Mastery	6	0	6
Need for Individualized support and time for practice	20	20	40

4.4.2.1 Learning Challenges and Obstacles

Drawing from the data presented in Table 22 complements this by quantifying the frequency of each code within its category, providing a clear view of how often specific challenges or support needs appeared in the dataset for Theme 1.

Table 22, learning challenges and obstacles emerged as the most frequent experiences reported by learners across both instructional approaches, totaling 109 occurrences, 49 in the TVHPI group and 60 in the CVHPI group. Every learner described encountering some form of difficulty when attempting to understand transformation geometry. These challenges were grouped into three categories: *struggling with geometry concepts*, *technical and resource challenges*, and *difficulties seeking help during the lesson.*

The issue of *struggling with geometry concepts* appeared more prominently among the CVHPI participants, with 38 instances compared to 16 in the TVHPI group. As one TVHPI learner explained, *"Reflections were easier for me, but when we started working with enlargements, I got lost because I did not understand the scale factors"* (FLA/TVHPI/R2). Similarly, a learner in the CVHPI group reflected, *"But rotations were hard for me when we had to apply them on the grid. More examples would have helped"* (FHA/VHPI/U2). The higher prevalence of this challenge within the CVHPI cohort indicates that traditional teaching methods may be less effective for certain learners. However, the presence of similar responses in the TVHPI group shows that technology-based instruction, while less frequently associated with this issue, does not fully eliminate it.

In contrast, *technical and resource-related challenges* were reported more often by TVHPI participants than by their CVHPI counterparts. For example, one learner remarked, *"The computer was slow, and the power went out while the teacher was explaining rotations, so I still found matrix transformations and rotations difficult"* (FLA/TVHPI/R1). Such instances highlight how the mode of delivery can influence the nature of obstacles faced, with technology-enhanced instruction introducing infrastructure-dependent problems.

Another less common, yet noteworthy, difficulty was *hesitation to seek help*. A TVHPI learner admitted, *"I got lost a lot and did not ask for help enough"* (FLA/TVHPI/U1), suggesting a reluctance to seek clarification during lessons. This reluctance, absent in the CVHPI accounts, may stem from the more individualistic nature of working with technology, which can sometimes foster a sense of isolation. This finding suggests the need for educators to foster environments, whether technology-enhanced or traditional, where learners feel comfortable requesting assistance and where collaboration and peer interaction are encouraged alongside individual work.

Taken together, these challenges point to the importance of *metacognition and self-regulated learning,* skills that several learners still identified as areas of weakness. As discussed in the subsequent

subsection, such skills are crucial for enabling students to take ownership of their learning, recognize when they require support, and adapt effectively to the demands of different instructional methods, including those that involve technological tools.

4.4.2.2 Metacognition and Self-Regulated Learning

In the *Metacognition and Self-Regulated Learning* category, the number of recorded instances was relatively small, 2 from the TVHPI group and 8 from the CVHPI group, yet these cases reveal how learners employed self-regulatory strategies to address their learning challenges. Such strategies were central to how they attempted to navigate difficulties during the learning process. The approaches they described included focusing attention, cultivating interest in the content, and engaging in self-reflection after lessons.

Within the TVHPI group, the two learners who mentioned this category stressed that maintaining concentration was critical to overcoming obstacles in geometry. They noted that focusing intently on tasks enhanced their understanding of the subject matter. As one participant shared, *"The shapes moved, but it did not always make sense to me. I think the learners who did better spent more time on the computer and paid more attention"* (FLA/TVHPI/U1). This illustrates how technology can foster cognitive engagement, thereby facilitating deeper comprehension of complex geometric transformations. However, this level of focus often depended on the learner's interest in the topic. In a similar vein, two CVHPI learners explained that their motivation to master geometry concepts helped them persist despite difficulties.

Self-reflection emerged more frequently among CVHPI participants. One learner explained, *"The teacher explained it, but sometimes I did not ask for more help; however, after class, I would think about what we did, which helped me understand better"* (MLA/VHPI/R1). This pattern suggests that in more traditional classroom settings, learners may need to take greater initiative in reviewing and processing

material independently, potentially because they lack the instant interactive feedback that is often available in technology-enhanced environments.

4.4.2.3 External Learning Support

This subsection explores the forms of assistance learners believed were crucial for effectively managing their learning. Out of 46 total references, 26 from the TVHPI group and 20 from the CVHPI group, it was evident that students in both instructional contexts required additional support to overcome their challenges. The main categories of assistance identified were *extensive use of GeoGebra*, *individualized teacher support*, and *time for ongoing practice*.

In the TVHPI group, six learners stressed that proficiency in transformation geometry was closely tied to sustained engagement with GeoGebra. Although the software facilitated learning, they emphasized that mastery still depended on repeated practice. As one learner noted, *"More time on the computer would have helped"* (FLA/TVHPI/R3), underscoring the importance of consistent and ongoing use of technological tools to deepen conceptual understanding.

Both groups also voiced a strong need for more personalized guidance from teachers and extended time for practice. This need appeared with equal frequency, 20 instances in each group, indicating that such limitations are shared across both instructional strategies. For example, a TVHPI participant explained, *"I think it would help if we had more time on the computer and more help from the teacher during the hard parts"* (FLA/TVHPI/R1), while a CVHPI learner similarly remarked, *"If the teacher gave me more time on that, it would have been better for me"* (MHA/VHPI/U2).

The equal occurrence of these requests highlights a common shortcoming in both approaches, pointing to the need for instructional designs that integrate longer, focused practice sessions with targeted

teacher feedback. This combination would better address learners' individual needs and strengthen their grasp of transformation geometry.

4.4.3 Instructional effectiveness and learner satisfaction.

This theme is built around five interlinked categories, beginning with the Role of Instructional Strategies in Learning, which serves as the foundation for Instructional Support and Guidance. From this base, the support is customized through Personalized Learning and Differentiated Instruction, enabling learners to receive targeted assistance tailored to their needs. This personalization encourages Practical Learning and Reinforcement, allowing students to apply their knowledge through hands-on activities that strengthen understanding. The final category, Learners' Expectations, evaluates how effectively these instructional strategies met or fell short of learners' anticipated outcomes.

Together, these categories capture the distinctive experiences of learners in both instructional groups, showing how each teaching approach shaped satisfaction and contributed to the learning process. The relationships between the theme, its categories, and the underlying codes are visually represented in Figure 23, while Table 23 presents the frequency of each code within its respective category, providing a detailed view of how often particular instructional elements were mentioned in the data.

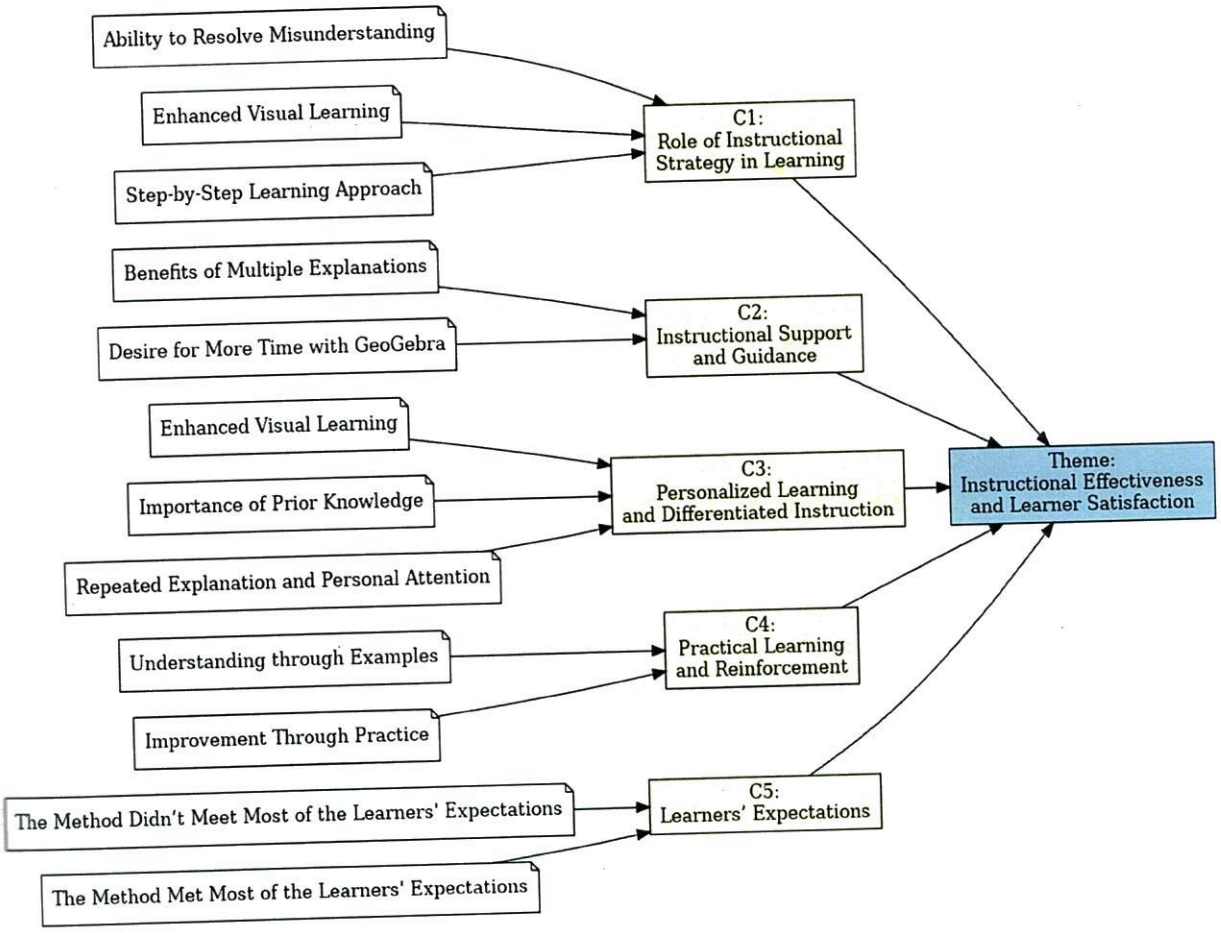


Figure 23: The emergence of Theme 2 from the categories and codes

Figure 23 shows the theme "Instructional Effectiveness and Learner Satisfaction" emerging from underlying categories and codes. The diagram illustrates how five categories, Role of Instructional Strategy in Learning, Instructional Support and Guidance, Personalized Learning and Differentiated Instruction, Practical Learning and Reinforcement, and Learners' Expectations, are grounded in specific learner responses (codes), which together shape the overarching theme of instructional effectiveness and learners' satisfaction.

Table 23: A count of codes and categories for Theme 2.

Row Labels	TVHPI	CVHPI	Total
Instructional Effectiveness and Learner Satisfaction	**147**	**125**	**272**
Role of Instructional Strategy in Learning	**91**	**47**	**138**
Ability to resolve misunderstanding	40	14	54
Enhanced Visual Learning	44	8	52
Step-by-Step Learning Approach	7	24	31
Instructional Support and Guidance	**8**	**2**	**10**
Benefits of Multiple Explanations	0	2	2
Desire for More Time with GeoGebra	8	0	8
Personalized Learning and Differentiated Instruction	**2**	**6**	**8**
Enhanced Visual Learning	1	0	1
Importance of Prior Knowledge	0	5	5
Repeated Explanation and Personal Attention	1	2	3
Practical Learning and Reinforcement	**21**	**46**	**67**
Understanding through examples	13	31	44
Improvement Through Practice	8	15	23
Learners expectations	**25**	**24**	**49**
The method did not meet most of the learners' expectations.	0	13	13
The method met most of the learners' expectations	25	11	36

4.4.3.1 Role of Instructional Strategy in Learning

As shown in Table 23, a total of 138 responses were recorded in this category, 91 from the TVHPI group and 47 from the CVHPI group, indicating that the instructional strategy had a notable influence on learners' outcomes and their engagement with geometry concepts. The roles identified under this category

included *resolving misunderstandings*, *enhancing visual learning*, and *adopting a step-by-step learning approach*.

Learners in the TVHPI group frequently reported that the instructional approach enabled them to resolve misunderstandings more effectively. One participant noted, *"Using GeoGebra helped me a lot because I could see the shapes change right in front of me. I understood what I had misunderstood"* (FHA/TVHPI/R1). In the CVHPI group, fewer learners expressed this benefit, though some still acknowledged progress through guided steps. As one learner explained, *"Reflections were easier for me, but when we moved on to rotations, they were confusing at first, but after following the steps, I saw some light"* (FLA/VHPI/R3). This pattern suggests that the interactive and visual elements of TVHPI, particularly via GeoGebra, enabled quicker identification and correction of errors compared to CVHPI.

The theme of *visual learning* was far more prevalent in TVHPI responses, appearing 44 times versus only 8 in CVHPI. A TVHPI participant remarked, *"GeoGebra made a big difference for me because I could see how the shapes were changing"* (FHA/TVHPI/R1). By contrast, CVHPI learners highlighted the more limited role of visual aids in traditional instruction, with one commenting, *"I remember one lesson where the teacher showed us how to use symmetry, and that helped me"* (FHA/VHPI/U3). These accounts reinforce the advantage of dynamic, real-time visualizations in TVHPI, while in CVHPI, structured progression served as the main method for guiding learners through each concept toward mastery.

Interestingly, the *step-by-step learning approach* was mentioned more often in CVHPI (24 occurrences) than in TVHPI (7 occurrences). This finding points to the structured nature of CVHPI, where Instructional Support and Guidance is a central feature for building learner confidence through carefully sequenced instruction. However, both approaches showed an ongoing reliance on teacher support, CVHPI through direct interaction, and TVHPI through explicit guidance to complement its visual tools.

4.4.3.2 Instructional Support and Guidance

Instructional support and guidance played an important role in shaping how learners experienced and evaluated their learning. In this category, there were 10 recorded instances, 8 from the TVHPI group and 2 from the CVHPI group, showing that the level and nature of support provided influenced how students perceived the effectiveness of their instructional method. This category was informed by two main codes: the desire for more time with GeoGebra and the benefits of multiple explanations.

In the TVHPI group, many learners expressed a strong wish for extended time working with GeoGebra, seeing it as essential to building a deeper understanding of transformation geometry. While the software offered clear benefits, learners felt that the allocated time was insufficient for mastery, which may have lowered satisfaction with the approach. One participant explained, *"I think I needed more time with GeoGebra to understand"* (MLA/TVHPI/R2). This aligns with findings from the earlier Challenges and Support Needs theme, where students emphasized that regular, sustained practice with GeoGebra was key to improving comprehension.

While TVHPI learners highlighted the role of additional time with technology, CVHPI learners placed greater value on teacher-led explanations and repeated examples as central to their progress. One CVHPI participant shared, *"The way the teacher taught us helped me get better because we practiced a lot"* (MHA/VHPI/U2). The ability of CVHPI instructors to revisit concepts in multiple ways appeared to positively influence learners' satisfaction.

These perspectives point to distinct strengths in each instructional strategy: TVHPI provided rich visual and interactive experiences that fostered independent learning, while CVHPI leveraged the adaptability of the teacher to offer varied and repeated explanations. Ultimately, both groups benefited from their respective methods: TVHPI learners from extended use of GeoGebra, and CVHPI learners

from teacher-led reinforcement, highlighting that effective instructional support depends on matching the strategy to the learners' needs.

4.4.3.3 Personalized Learning and Differentiated Instruction

This category underscores the value of tailoring instruction to address the unique needs of individual learners. With a total of 8 references, 2 from the TVHPI group and 7 from the CVHPI group, it is evident that CVHPI created more frequent opportunities for personalized and differentiated learning experiences.

The main elements within this category were *repeated explanations*, *personal attention*, and *the role of prior knowledge*. For instance, one CVHPI learner described how direct teacher intervention clarified uncertainties about rotation, reflecting the immediate benefits of one-on-one support. In contrast, such customized assistance appeared less regularly in TVHPI, suggesting that while technology offers rich interactive features, it does not fully substitute for direct instructor interaction in helping learners develop a complete understanding of concepts.

This reliance on human attention and repeated clarification is closely linked to the significance of prior knowledge. Learners often require targeted help to bridge the gap between their existing understanding and new geometric concepts. In this way, CVHPI's emphasis on repeated explanations and direct engagement with learners demonstrates its strength in providing adaptive, learner-centered support that complements the structured delivery of new content.

4.4.3.4 Practical Learning and Reinforcement

Practical learning and reinforcement emerged as pivotal elements in shaping how learners experienced the lessons and evaluated the instructional strategies. This category was recorded 67 times in total, 21 from

the TVHPI group and 46 from the CVHPI group, indicating that while both approaches emphasized it, it was particularly prominent in CVHPI. Two main codes defined this category: the role of examples and the role of practice in improving understanding.

Examples played a vital role in strengthening comprehension, with CVHPI learners mentioning this benefit more frequently. The higher number of occurrences in the CVHPI group suggests that examples may have a more pronounced effect in traditional settings, likely because of the direct, guided nature of the instruction. Nevertheless, TVHPI also demonstrated strength in this area, especially when examples were delivered through the interactive, visual features of GeoGebra. This reinforces the idea that both strategies provide meaningful reinforcement, albeit through different channels, and links closely linked to the broader theme of instructional support and guidance.

The code *"improvement through practice"* appeared equally in both groups, with eight references each. One TVHPI learner reflected, *"I thought it would be confusing, but GeoGebra made everything clearer for me, especially with enlargements after practicing"* (FHA/TVHPI/U3). Similarly, a CVHPI participant stated, *"I found reflections easier because we practiced them a lot"* (MLA/VHPI/R3). The parallel emphasis on practice in both instructional strategies underlines the value of consistent reinforcement for mastering geometry concepts. Whether achieved through technology-based activities or more traditional exercises, learners recognized that repeated practice was central to their progress and understanding.

4.4.3.5 Learners' Expectations

This category explores the alignment between instructional strategies and learners' expectations for studying geometry. Across 49 recorded instances, 25 from the TVHPI group and 24 from the CVHPI

group, students shared mixed views on how well their learning experiences matched what they had anticipated. Two distinct codes emerged: *"The Method Met Most of the Learners' Expectations"* and *"The Method Did Not Meet Most of the Learners' Expectations"*.

In the TVHPI group, all 25 references indicated that the instructional method met or exceeded expectations. Learners frequently described being positively surprised by their experiences. As one participant shared, *"I did not think I would enjoy using the computer for geometry, but I was surprised at how much fun it was"* (FHA/TVHPI/R2). Likewise, a CVHPI learner reflected, *"I thought reflections would be the most confusing part, but I was surprised at how fast I picked them up"* (FHA/VHPI/U1). The higher rate of satisfaction in the TVHPI group suggests that the integration of technology not only enhanced understanding but also aligned more closely with expectations for a modern, interactive classroom. This resonates with earlier findings on the benefits of visual learning and the capacity to resolve misunderstandings, both of which likely contributed to meeting learners' expectations.

On the other hand, while CVHPI was effective in many respects, it appeared to fall short for learners who anticipated a more dynamic or participatory style of instruction. Several students in this group expressed that their experiences did not fully match what they had hoped for. One participant admitted, *"I did not think I would be so confused with symmetry. I expected it to be simpler, but I kept making mistakes when figuring out where the lines went. It surprised me how much more attention I needed to pay"* (FLA/VHPI/U1). Such feedback reveals that some CVHPI learners felt their instructional approach did not deliver the level of engagement or clarity they expected, echoing challenges documented earlier about understanding transformation geometry.

4.4.4 Discussion and Summary of the Findings on Learners' experiences and understanding within TVHPI and CVHPI

4.4.4.1 Discussion

This discussion engages with the research questions underpinning the fourth objective by exploring the *challenges* and *support needs* encountered in both Technology-Enhanced Van Hiele Phased Instruction (TVHPI) and Conventional Van Hiele Phased Instruction (CVHPI). It also examines learners' *perceptions of the effectiveness* of these two instructional approaches in supporting their understanding of transformation geometry. In doing so, the discussion situates the findings within established theoretical frameworks, connects them to existing literature, and draws out practical implications for the design and delivery of instructional strategies in geometry education.

The results reveal that both TVHPI and CVHPI present challenges for learners, though the nature and causes of these difficulties differ substantially. In line with previous research, CVHPI learners reported greater struggles with conceptual understanding, particularly when engaging with abstract transformations such as rotations and scaling. These findings resonate with aids Açıkgül (2022), who noted that traditional instruction, while structured, can limit the development of geometric reasoning when it relies primarily on static representations. Without dynamic visual aids, learners must expend significant cognitive effort in mentally simulating transformations, an effort that may overwhelm working memory and hinder deeper conceptual insight.

By contrast, TVHPI learners rarely identified conceptual difficulty as their primary challenge. Instead, their concerns centered on technical and infrastructural constraints, such as unreliable electricity, limited access to computers, and insufficient time to explore GeoGebra. These obstacles are consistent with findings from Adelabu et al. (2022), Afshari et al. (2009), and Roxana (2019), who stress that technology-enhanced learning environments are only as effective as the resources and infrastructure supporting them. Furthermore, this aligns with Moore et al. (2018) on the *digital divide*, which emphasizes that inequities in technological access can inadvertently widen achievement gaps, especially in resource-limited contexts.

The implication here is that while TVHPI offers strong pedagogical affordances, these benefits may be undermined if the supporting infrastructure is unstable or learners are given limited exposure to the technology. This reinforces the need for policy and school-level investment in reliable power supply, adequate device provision, and dedicated instructional time for software use.

Despite infrastructural constraints, TVHPI learners consistently emphasized the benefits of enhanced visual learning. GeoGebra's interactive capabilities allowed them to manipulate shapes, observe transformations in real time, and integrate both procedural and conceptual learning. This experiential engagement facilitated smoother progression through the Van Hiele levels, aligning well with cognitive load theory, which posits that reducing extraneous mental load enables learners to focus on understanding core principles (Plass et al., 2010). In transformation geometry, where visualization is critical, reducing the cognitive demand of imagining transformations frees learners to reason about their properties and implications.

The ability to *see* transformations rather than simply *infer* them may also explain why TVHPI learners reported greater satisfaction with their learning experiences. This finding echoes Sunzuma (2023), who observed that dynamic visualization tools bridge the gap between abstract concepts and learner comprehension, particularly in spatially demanding topics such as rotations and reflections.

A notable finding from both groups was the strong demand for individualized support and extended practice. Regardless of method, learners valued repeated exposure to problems, teacher-led clarification of misconceptions, and structured opportunities to consolidate learning. This reinforces the principle, found across mathematics education literature, that scaffolding is a universal need for mastering complex topics (Hattie, 2009).

However, TVHPI learners reported a reluctance to seek help, sometimes feeling isolated when working with technology. This contrasts with studies such as Zheng et al. (2022), which suggest that digital tools can promote collaboration. One possible explanation is that in the implementation studied

here, technology use was individualized rather than collaborative, reducing opportunities for peer discussion. This points to the need for integrating cooperative learning strategies within technology-enhanced settings to counteract potential feelings of isolation.

Learners' expectations for transformation geometry lessons were shaped by their prior knowledge, exposure to the curriculum, and earlier learning experiences. Many entered lessons anticipating the opportunity to visualize transformations, manipulate figures, and receive explicit, step-by-step guidance expectations largely informed by earlier experiences with static diagrams and formulaic exercises.

In this respect, TVHPI was consistently praised for delivering on these expectations. The capacity to see transformations in real time not only aligned with learners' preconceptions of what an engaging geometry lesson should be, but also deepened their conceptual understanding. In contrast, CVHPI learners valued the predictability and structure of their lessons, with many praising the teacher's ability to provide multiple explanations and targeted feedback. This aligns with Larbi (2021), Machisi & Feza (2021), Moru et al. (2021), and Tahani (2016), who found that structured instructional methods remain effective for learners who depend on clear procedural guidance.

However, CVHPI did not always meet the expectations of learners who desired more dynamic, participatory experiences. In some cases, this misalignment led to frustration or disengagement. From an instructional design standpoint, this suggests that even within conventional approaches, integrating interactive elements and visual supports could help maintain engagement and meet evolving learner expectations.

Overall, the evidence underscores the *complementary strengths* of TVHPI and CVHPI. TVHPI excels in conceptual development through visual and interactive learning, while CVHPI supports procedural mastery through structured explanation and teacher scaffolding. Neither approach alone fully addresses all learner needs, suggesting that a blended instructional model could be particularly effective.

4.4.4.2 Summary

The qualitative findings point to clear distinctions in learners' experiences and conceptual development under the two instructional strategies, Technology-Enhanced Van Hiele Phased Instruction (TVHPI) and Conventional Van Hiele Phased Instruction (CVHPI). TVHPI was frequently linked to heightened engagement, richer visual learning opportunities, and stronger alignment between instructional delivery and learners' expectations. In contrast, CVHPI was characterized by a structured, sequential approach that offered more consistent personalized support and encouraged deeper post-lesson reflection. Across both approaches, learners underscored the centrality of sustained practice, targeted teacher guidance, and the pursuit of conceptual clarity. However, the nature of the support provided, and the degree of satisfaction with the learning process, varied notably between the two instructional contexts.

CHAPTER FIVE

SYNTHESIS OF FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

This chapter provides an integrated interpretation of the study findings and outlines their broader meaning about the research objectives. Unlike Chapter 4, which presented and discussed results for each objective separately, this chapter synthesizes the findings to highlight relationships, patterns, and implications that emerge across the objectives. The chapter also presents the conclusions derived from the study, followed by theoretical and practical implications, recommendations for practice, policy, and further research, as well as the limitations of the study. In addition, this chapter introduces the Geometry Pedagogical Improvement Cycle (GeoPIC), a framework developed from the study's findings to guide effective instructional practice in geometry.

5.1 Synthesis of Key Findings

This section integrates and interprets the results from the four research objectives, highlighting how instructional strategies, learners' attitudes, and conceptual development through Van Hiele levels interact within the context of teaching transformation geometry.

5.1.1 Influence of Instructional Strategies on Learners' Attitudes and Van Hiele Levels

The study found that both TVHPI and CVHPI significantly improved learners' attitudes toward transformation geometry and their progression through the Van Hiele levels. However, TVHPI consistently produced greater gains in both attitudes and conceptual reasoning levels. This confirms that

while structured, phased instruction fosters conceptual development, the integration of technology amplifies learner engagement and understanding. The enhanced visualization and interactive features offered by tools like GeoGebra contributed to deeper conceptual exploration, making technology a valuable complement to traditional instructional methods.

5.1.2 Role of Geometry Attitudes in Learner Progression

Although learners' attitudes toward geometry improved significantly, their predictive and mediating roles in progression through Van Hiele levels were limited. Neither initial attitudes nor post-instruction attitudes significantly predicted conceptual advancement. Attitude change during instruction did not mediate the effects of instructional strategy, though a marginal moderation effect indicated that attitude change might have some influence within the conventional instructional setting. This suggests that affective factors, while important outcomes themselves, do not strongly drive conceptual growth in geometry but may play context-dependent roles.

5.1.3 Contribution of Technology to Engagement and Conceptual Understanding

The integration of GeoGebra within the Van Hiele Phased Instruction provided learners with meaningful opportunities for visualization, experimentation, and self-regulated learning. Qualitative data supported these quantitative results by showing that learners exposed to TVHPI reported higher motivation, autonomy, and engagement compared to those in the CVHPI group. This synergy between structured instructional design and digital tools aligns with competency-based curricula goals, emphasizing learner-centered and active engagement approaches to mathematics education.

5.2 Study Conclusions

This study examined the impact of instructional strategies on learners' attitudes and progression through the Van Hiele levels of geometric reasoning in transformation geometry. Situated within the Van Hiele theory and the cognitive constructivist paradigm, the findings highlight the pivotal role of structured instruction and learner engagement in promoting conceptual growth.

The results showed that CVHPI and TVHPI significantly improved learners' attitudes and Van Hiele levels. However, TVHPI was consistently more effective in promoting positive attitude shifts and higher-level reasoning. These results affirm the Van Hiele model, which posits that learners progress through hierarchical levels of understanding when instruction is appropriately sequenced. The superiority of TVHPI also resonates with constructivist theory, which emphasizes active knowledge construction through learner-centered, interactive experiences epitomized by GeoGebra's dynamic geometry environment.

While learners' attitudes improved significantly, they did not predict or mediate progression through the Van Hiele levels. This suggests that although affective dispositions enrich the learning environment, cognitive development in geometry is more strongly driven by the nature of instruction and learners' conceptual starting points, consistent with Van Hiele's view of stage-based cognitive readiness. The significant predictive value of pre-instruction Van Hiele levels further highlights the importance of diagnostic teaching aligned with cognitive readiness.

The GeoPIC framework emerged as a key outcome from this synthesis, providing a reflective, cyclical structure for teachers to improve geometry instruction. It operationalizes Van Hiele and constructivist principles by guiding teachers through adaptive phases of planning, execution, and continuous refinement, ensuring alignment between learner needs and instructional design. Ultimately, this study demonstrates that impactful geometry learning occurs where cognitive scaffolding, affective support, and responsive pedagogy converge.

5.3 Evolution and Refinement of the Geometry Pedagogical Improvement Cycle (GeoPIC)

5.3.1 Evolution of the GeoPIC Framework

During the progression of this study, an original framework, the Geometry Pedagogical Improvement Cycle (called Ndungo framework) was conceptualized to guide geometry instruction improvement. This initial framework was published in Ndungo et al. (2025)[11]. Following full data analysis and synthesis of findings, the framework was refined and expanded to address observed gaps and integrate new insights. The refined version was published in Ndungo et al. (2025a)[12] and is presented in this subsection as a major contribution of the thesis.

This framework, developed as a core outcome of this thesis, offers a systematic instructional model for teaching transformation geometry. Drawing from empirical data, established theory, and pedagogical insights, GeoPIC provides a structured, adaptable cycle that supports learners' progression through the Van Hiele levels of geometric reasoning. The original GeoPIC emerged from the initial phases of this study, identifying that learners benefit most when instruction is adaptive, phased, and responsive to their conceptual readiness. Conventional Van Hiele Phased Instruction ensured structured progression, while Technology-Enhanced Van Hiele Phased Instruction enriched learner engagement via dynamic tools such as GeoGebra. However, challenges, including sensitivity to dropout in TVHPI and resource constraints in CVHPI, signaled the need for a more hybrid, flexible, and teacher-guided

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[11] Ndungo, I., Balimuttajjo, S., & Akugizibwe, E. (2025). Towards Improved Geometry Instruction: Learners' Experiences with Conventional and Technology-Enhanced Van Hiele Phased Instruction. *The Electronic Journal for Research in Science & Mathematics Education.* 29(2), 33-56. https://ejrsme.icrsme.com/article/view/23815

[12] Ndungo, I., Balimuttajjo, S. & Akugizibwe, E. (2025). Refining and expanding the geometry pedagogical improvement cycle (GeoPIC): A conceptual contribution to competency-based geometry instruction. *Journal of Research Innovation and Implications in Education, 9*(3), 508 – 517. https://doi.org/10.59765/jriie.9.3.47

framework. Consequently, this thesis elaborated and expanded GeoPIC into six distinct, interrelated phases, forming a continuous cycle of instructional planning, implementation, and reflection.

5.3.2 Refinement and Expansion of GeoPIC Framework

The refinement of the GeoPIC framework within this thesis was driven by empirical insights and critical reflection on instructional practices observed during the study. While the initial framework provided a coherent model, it became clear that explicit integration of information and communication technology was essential to reflect contemporary teaching realities.

Additionally, the role of learners' attitudes toward geometry, thoroughly examined in this thesis, underscores the necessity of embedding affective dimensions within the instructional cycle (Ndungo, Balimuttajjo, Akugizibwe, et al., 2025). Furthermore, alignment with the Competency-Based Curriculum's emphasis on formative assessment and learner-centered progression required strengthening iterative reflection and adjustment mechanisms within the model. Through these enhancements, the refined GeoPIC framework offers a more comprehensive, contextually relevant, and practically useful guide for teachers, learners, and curriculum developers striving to improve geometric reasoning instruction in diverse secondary school settings.

The GeoPIC framework is structured around six interrelated phases that form a continuous cycle of instructional planning, implementation, and reflection. Each phase is intentionally designed to support learners' progression through the Van Hiele levels of geometric reasoning while remaining adaptable to different classroom environments, learner profiles, and curricular expectations. Figure 24 illustrates the six phases of the framework, followed by a detailed explanation for each phase.

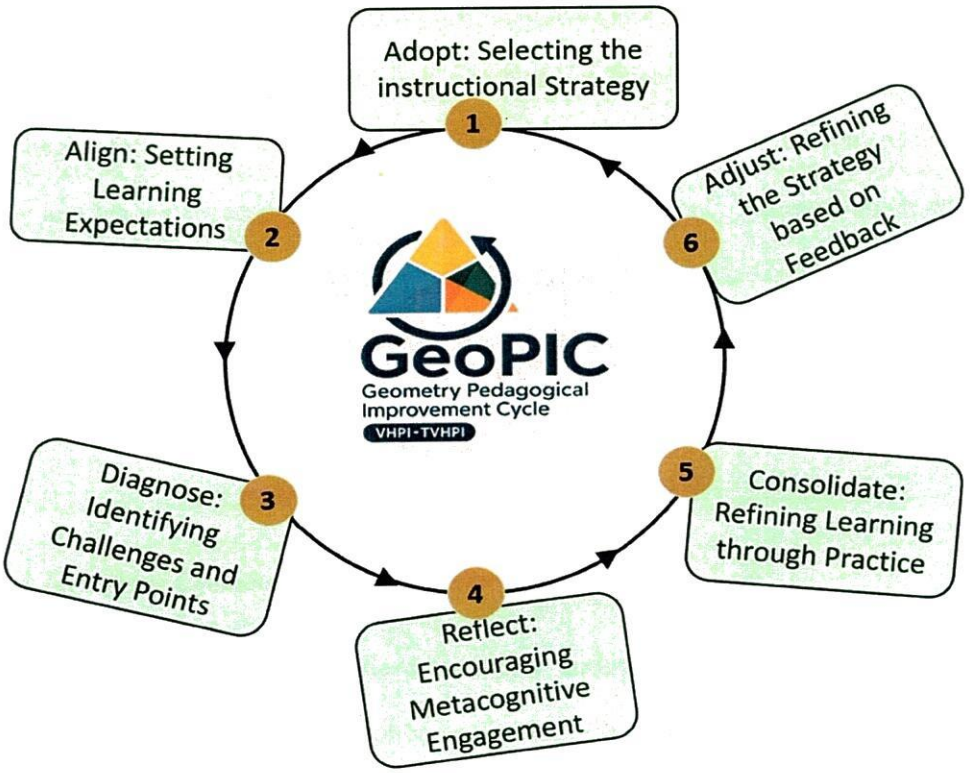


Figure 24: Showing the phases of the Geometry Pedagogical Improvement Cycle (GeoPIC-Framework)

1. Adopt: Selecting the Instructional Strategy

This phase focuses on instructional decision-making. The teacher determines whether to apply Conventional Van Hiele Phased Instruction, which emphasizes hands-on activities using tools like graph paper, mirrors, and rulers, or to adopt Technology-Enhanced Van Hiele Phased Instruction, which integrates digital platforms such as GeoGebra for dynamic visualization. The selection is informed by learners' prior experiences, resource availability, and the complexity of the transformation geometry concepts to be addressed. While not part of the Van Hiele phases themselves, this strategic adoption ensures that instruction begins from an informed and purposeful foundation.

2. Align: Setting Learning Expectations

Once the strategy is selected, the teacher communicates clear learning goals and assessment criteria. This phase aligns with the 'Information' phase of Van Hiele, where learners are introduced to the topic and begin forming connections (Vojkuvkova, 2012). The GeoPIC framework emphasizes transparency and learner ownership at this stage. Students are not only informed about what they will learn, but also how they will engage with the material and how their understanding will be evaluated. In TVHPI contexts, this might include introducing students to the GeoGebra interface and setting up expectations for independent exploration.

3. Diagnose: Identifying Challenges and Entry Points

Here, teachers conduct diagnostic assessments through observation, formative pre-tests, and discussions to identify learners' misconceptions and determine their current Van Hiele level. This stage builds upon the Van Hiele model by explicitly acknowledging that learners enter instruction at different stages of reasoning. Rather than assuming uniform readiness, GeoPIC equips teachers to tailor content and support based on real-time understanding of learners' needs. In doing so, it reinforces the Directed Orientation phase by shaping the experiential activities to match learner readiness (Crowley, 1987; Larbi, 2021; Naufal et al., 2021).

4. Reflect: Encouraging Metacognitive Engagement

In this phase, learners engage in self-reflection, peer discussions, and explanation of thought processes, often through journaling, guided questions, or debriefing sessions after a GeoGebra activity. Teachers also reflect on instructional choices and student responses, identifying what worked and what needs revision. This mirrors and enriches the Explication phase in Van Hiele's model by fostering metacognition and reinforcing the use of geometric language, both of which are crucial for developing deductive reasoning (Moru et al., 2021; Vojkuvkova, 2012).

5. Consolidate: Reinforcing Learning through Practice

This phase corresponds to the Free Orientation and Integration phases of Van Hiele, during which learners begin to solve more complex tasks and consolidate their understanding of the material. Under GeoPIC, consolidation is structured through varied application tasks, paper-based constructions, digital manipulations, real-life modeling, and assessments. This stage is crucial for learners to apply transformations across various contexts and to demonstrate a transition from procedural execution to conceptual understanding (Eisenhart et al., 1993; Mills, 2016; Ndungo, Balimuttajjo, & Akugizibwe, 2025a).

6. Adjust: Refining the Strategy Based on Feedback

Teachers continue improving instruction even after consolidation. In this final phase, teachers analyze learning outcomes and adjust strategies for subsequent cycles. This stage is not explicitly addressed in Van Hiele's original model but is vital in today's competency-based and differentiated instruction contexts. Adjustments may involve re-grouping learners, revisiting challenging concepts, or integrating new digital tools. It parallels the continuous improvement loop found in professional development models such as Plan-Do-Check-Act (Pratik & Vivek, 2017).

5.3.3 Practical Application of the GeoPIC Framework in Classroom Instruction

A practical illustration of GeoPIC in a geometry classroom involves teaching reflection across the y-axis through its six phases. The teacher first adopts a blended approach using both traditional tools and GeoGebra, then aligns expectations by clarifying learning outcomes. During the diagnose phase, misconceptions such as confusing axes are identified and addressed. Learners then reflect on their reasoning through discussion and self-assessment, promoting metacognition.

In the consolidation phase, additional practice reinforces understanding and builds confidence. Finally, the adjust phase involves reviewing outcomes and refining future lessons. This cyclical approach ensures adaptive, learner-centered instruction that supports progression through Van Hiele levels and addresses classroom realities.

5.4 Limitations of the study

While this study generated valuable insights into mathematics education and geometry instruction, several limitations should be considered when interpreting the findings.

First, the quasi-experimental design relied on intact classes without full randomization, introducing potential selection bias; for example, schools already predisposed to technology use could influence results. Although matched sampling, pretesting, and inclusion of covariates in DiD and GOL models helped mitigate internal validity threats, some residual bias cannot be ruled out.

Second, the study's Ugandan context limits generalizability. Local conditions such as large class sizes, irregular power supply, limited computer access, and varying teacher ICT training differ significantly from other regions, which may affect the scalability of TVHPI.

Third, self-reported attitude measures may be influenced by social desirability bias, and learner dispositions like self-efficacy or long-term motivation were beyond the study's scope. Future studies could integrate self-regulated learning frameworks to capture these broader cognitive and affective processes.

Lastly, despite teacher training and co-developed lesson plans, variations in instructional delivery (pacing, engagement, or technology use) may have affected learners' experiences and progression.

5.5 Implications for Mathematics Education

The findings of this study carry several important implications for the theory and practice of mathematics education, particularly in the teaching and learning of geometry.

First, the study confirms the critical role of instructional design in shaping learners' attitudes and cognitive progression through Van Hiele levels. The stronger performance of the Technology-Enhanced Van Hiele Phased Instruction (TVHPI) demonstrates that technology integration, when guided by structured pedagogical models, enhances conceptual understanding, engagement, and attitudinal outcomes. This finding supports calls for more significant investment in digital tools, such as GeoGebra, within mathematics classrooms and teacher training programs.

Second, the study reaffirms the Van Hiele Theory's hierarchical model of geometric reasoning. Learners advanced progressively through the levels when instruction was appropriately sequenced and cognitively aligned. The implication is that teachers must be trained to recognize and respond to learners' current Van Hiele levels, ensuring instruction is neither too advanced nor too elementary for meaningful progression.

Third, the findings suggest that positive attitudes are valuable but do not directly drive geometric reasoning gains. This implies that affective and cognitive domains must be addressed distinctly in classroom practice. Teachers should nurture a supportive learning environment and maintain intentional scaffolding of conceptual understanding aligned with the cognitive constructivist model.

Finally, introducing the GeoPIC framework offers a replicable and reflective structure for instructional improvement. GeoPIC encourages mathematics educators to cycle through adopting, aligning, diagnosing, reflecting, consolidating, and adjusting their pedagogical strategies. This positions teachers as active designers of instruction, responsive to both data and learner feedback, and promotes a culture of continuous improvement.

Together, these implications emphasize the need for curriculum reforms, teacher professional development, and classroom practices that recognize geometry as both a content area and a structured cognitive journey that thrives under dynamic, well-sequenced, and learner-responsive instruction.

5.6 Recommendations

5.6.1 Recommendations for Practices

Mathematics teachers should consider adopting the Geometry Pedagogical Improvement Cycle framework developed in this study. This six-phase cycle, Adopt, Align, Diagnose, Reflect, Consolidate, and Adjust, offers a structured approach for guiding instruction in transformation geometry. The framework emphasizes adaptive, conceptual diagnosis and continuous improvement, aligning well with the Van Hiele Theory and Constructivist principles of learner-centered engagement.

Given the demonstrated effectiveness of Technology-Enhanced Van Hiele Phased Instruction, particularly in advancing learners' reasoning from visualization to abstraction and deduction, teachers are encouraged to integrate dynamic tools like GeoGebra into their instructional practices.

Differentiated instruction based on learners' initial Van Hiele levels should also be emphasized, with early diagnostic assessment helping teachers scaffold support for those starting at lower reasoning levels. Though all participating teachers were trained and lesson plans were collaboratively designed, some variability in delivery persisted, highlighting the need for peer coaching, co-teaching, and reflective practice to ensure fidelity and effectiveness in implementation.

5.6.2 Recommendations for Policy

Curriculum developers and education policy makers should consider incorporating Van Hiele-based instructional approaches within national mathematics curricula, particularly for geometry topics. The phased structure of the Van Hiele theory progressively facilitates cognitive development, and its alignment with competency-based learning makes it suitable for curriculum reform efforts.

Professional development programs should include specialized modules for geometry instruction using ICT tools, such as GeoGebra. Institutions such as teacher training colleges and national education institutes should formally adopt the GeoPIC model into teacher training syllabi and resource guides.

Furthermore, standard assessment policies should be revised to reflect Van Hiele levels, allowing schools to measure learners' conceptual growth in ways that go beyond procedural correctness.

5.6.3 Recommendations for Further Research

While this study revealed important insights into instructional strategies and learners' progression, several areas warrant further investigation. First, future research should explore the longitudinal impact of TVHPI and the GeoPIC model, assessing whether improvements in Van Hiele levels are sustained over time and transfer to other mathematical domains.

Second, studies should analyze learning gains across specific Van Hiele levels to determine which levels benefit most from CVHPI and TVHPI, particularly at higher reasoning levels. Research should also examine topic-specific outcomes within transformation geometry, such as reflections, rotations, translations, and enlargements, to identify which concepts gain the most from technology-enhanced instruction.

Third, future studies should consider learner subgroups, including gender, school location (rural vs. urban), and prior knowledge, to provide insights for differentiated instruction and equity-focused interventions.

Fourth, although learners' attitudes improved, attitude change did not mediate Van Hiele progression. This invites investigation into other psychological and cognitive factors, including spatial reasoning, classroom environments, and teacher-related variables such as beliefs and pedagogical content knowledge.

Finally, replicating this study in different socio-cultural or educational contexts and testing the GeoPIC framework in other areas of mathematics (e.g., algebra or statistics) would enhance the generalizability and versatility of the findings.

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APPENDICES

1.1 Transformation Geometry Achievement Pre-Test

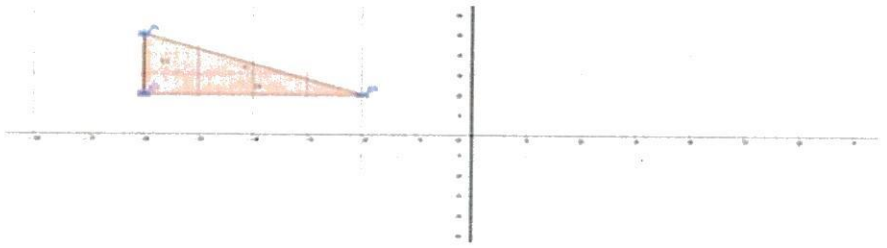
Student ID (compulsory) ..... Name: .....

School ID (compulsory) .....

Write your responses in the space provided

Level 1

1. Reflect the triangle ABC across the line $y = x$. What are the coordinates of the reflected image?

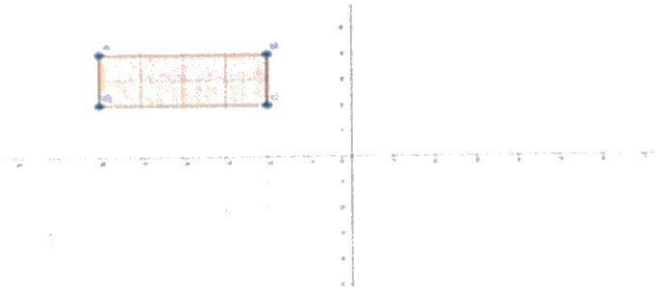


2. Distinguish between clockwise and anticlockwise rotation for a square by describing how its vertices move during each type of rotation.

3. Draw the lines of symmetry for an equilateral triangle. How many are there?

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4. Reflect the given square over the y-axis.



5. Draw two pairs of squares that are similar.

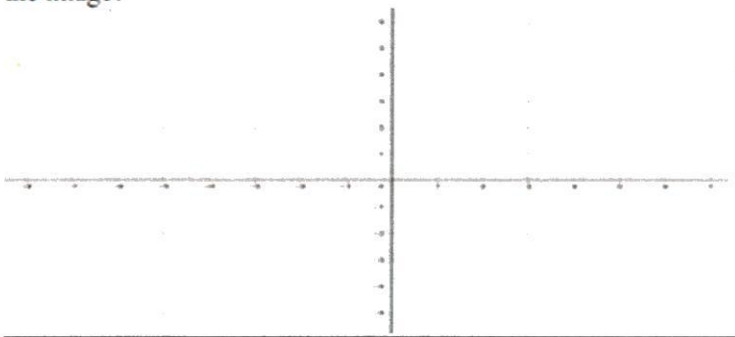
Level 2

6. On a graph, plot the point P(2, -1). Rotate it counterclockwise by 90 degrees about the origin. Plot the new point and label it as P'

7. Plot the line y = 3 on a Cartesian plane. Reflect the line through the x-axis and write down the equation of the image line.

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8. Given a point A(2, -3), perform a reflection over the x-axis. What are the coordinates of the image?



9. Two triangles, ABC and DEF, are similar. If AB = 4 cm and DE = 6 cm, and the ratio of their corresponding sides is 3:2, what is the length of side EF?

10. An enlargement with a scale factor of 2 is applied to a rectangle with vertices at (1, 1), (1, 3), (3, 3), and (3, 1). Find the coordinates of the vertices of the enlarged rectangle.

Level 3

11. A square object with a side length of 5 units undergoes a dilation with a linear scale factor of 3. Calculate the area of the original square and the area of the enlarged square.

12. Determine the transformation matrix for a dilation with a linear scale factor of 0.5.

13. If the determinant of a transformation matrix is 4, what is the corresponding area scale factor?

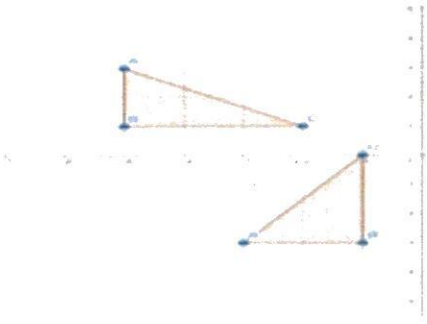
14. A triangle has vertices at (1,1), (1,4), and (4,1) on a coordinate grid. If it is dilated with a linear scale factor of 2, calculate the area of the original triangle and the area of the dilated triangle.

15. Given an object with vertices at (2,2), (2,4), (4,4), and (4,2) on a coordinate grid, and a transformation matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ plot the image of the object after the transformation.

Level 4

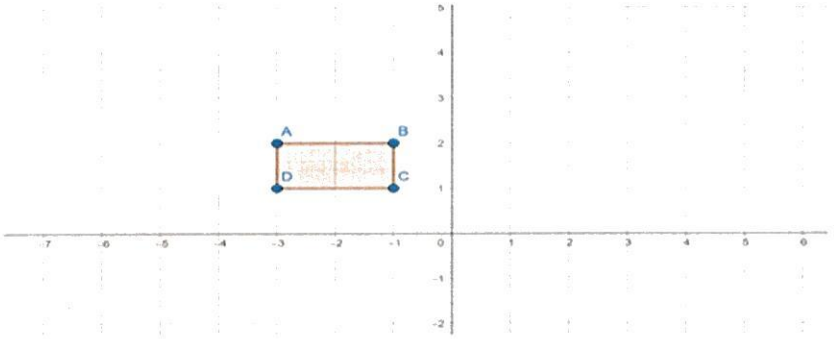
16. Given the transformation matrix $A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$, calculate the resultant matrix AB. Illustrate on a coordinate grid how this results matrix affects the vector (1,2).

17. Triangle ABC was rotated to triangle $A'B'C'$ as shown below. Find the angle and center of rotation.





18. Given a composite transformation involving a rotation of 45 degrees, center (0, 0) followed by a reflection over the y-axis, derive the single transformation matrix representing this composite transformation. Use it to transform the square $ABCD$ and plot $A'B'C'D'$ and $A''B''C''D''$ on the coordinate grid below.



19. Using the concept of similarity, derive the formula for the area of an enlarged figure. Apply this to enlarge a triangle with an area of 10 cm^2 by a scale factor of 3.

20. Given two similar triangles with side lengths in the ratio 3:4, prove that their corresponding medians are also in the same ratio. Illustrate this with a diagram of the triangles and their medians.

1.2 Transformation Geometry Achievement Post-Test

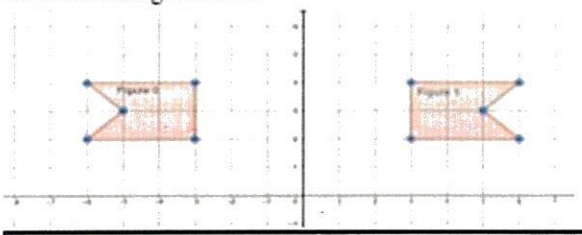
Students ID (compulsory) ..... Name.....

School ID (compulsory) .....

Write your responses in the space provided

Level 1

1. Name the type/form of transformation used to obtain Figure 1 from Figure 0 as shown on the coordinate grid below.

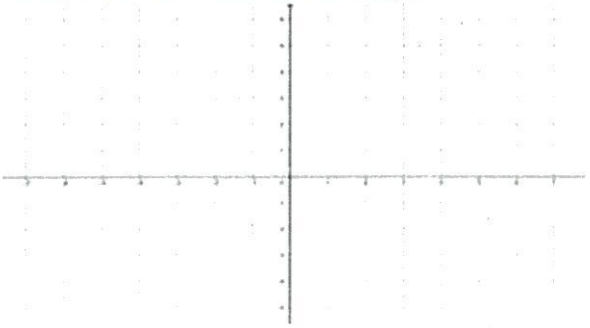




2. If a shape has rotational symmetry of order 4, what does that mean about the possible rotations it can undergo?

3. Which of these transformation(s) change the size of a shape: reflection, rotation, or enlargement?

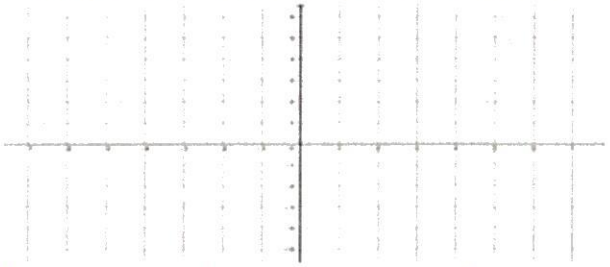
4. Reflect the shape given by the points $A(1,1), B(4,1), and\ C(4,3)$ across the x-axis. What are the coordinates of the reflected image?



5. What happens the coordinate (x, y) when reflected through the $y - axis$?

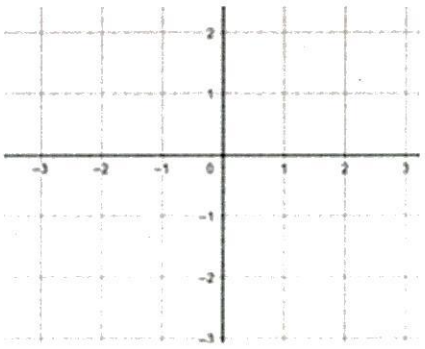
Level 2

6. A figure is rotated counterclockwise by 45 degrees around the origin. If point $P(2\ 2)$ is on the figure, what are its new coordinates after rotation?

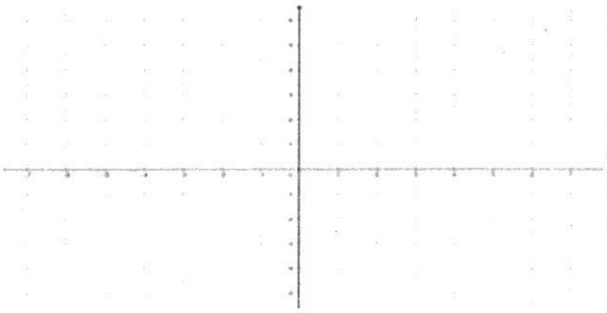




7. Determine the transformation matrix for a reflection over the line $y = 0$.

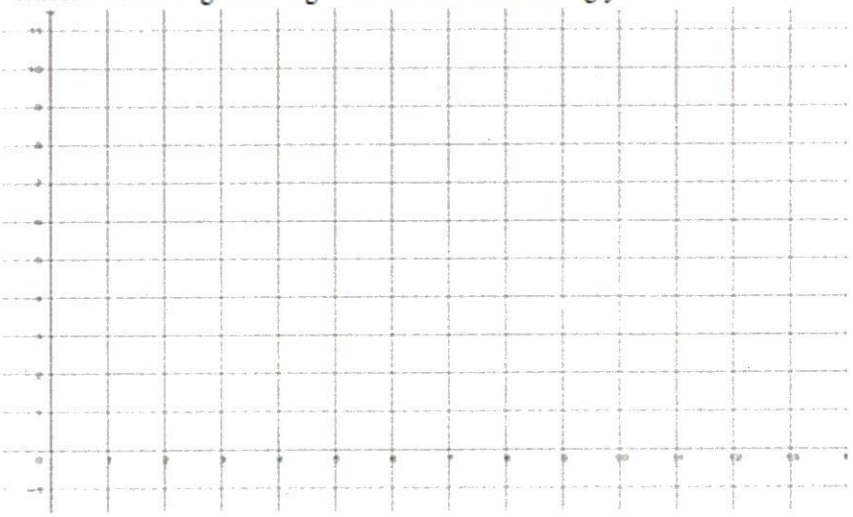


8. Plot the points $A(2, 3)$ *and* $B(4, 5)$ on a Cartesian plane. Perform a reflection over the x-axis, followed by another reflection through the line $= x$. Plot the image points and label them as A' *and* B' and A'', and B''.



9. Given two similar triangles ABC and DEF, where $AB = 6\,cm, BC = 8\,cm, DE = 9\,cm$, and $EF = 12\,cm$, plot these triangles on a grid. Determine the scale factor of the enlargement required to transform triangle ABC into triangle DEF.

10. Draw a rectangle with vertices at $A(1,1), B(1,3), C(3,3)$, and $D(3,1)$ on a Cartesian plane. Perform an enlargement with a scale factor of 2 centered at the origin. Plot the vertices of the enlarged rectangle and label them accordingly.



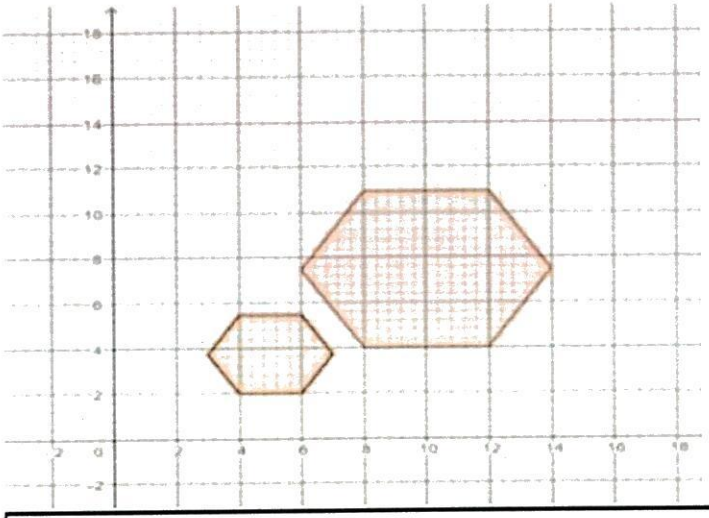


Level 3

11. Find the inverse of the transformation matrix $\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix}$

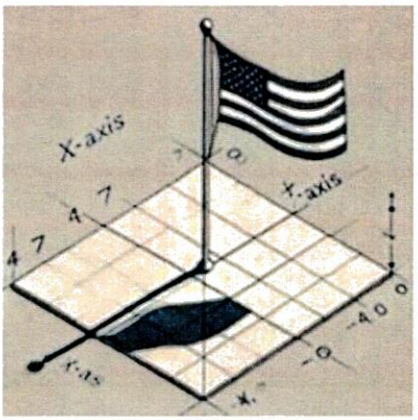
12. A square object with a side length of 10 units undergoes a dilation with a linear scale factor of 2. Calculate the area of the original square and the area of the enlarged square.

13. Find the scale factor for the transformation illustrated on the diagram below.

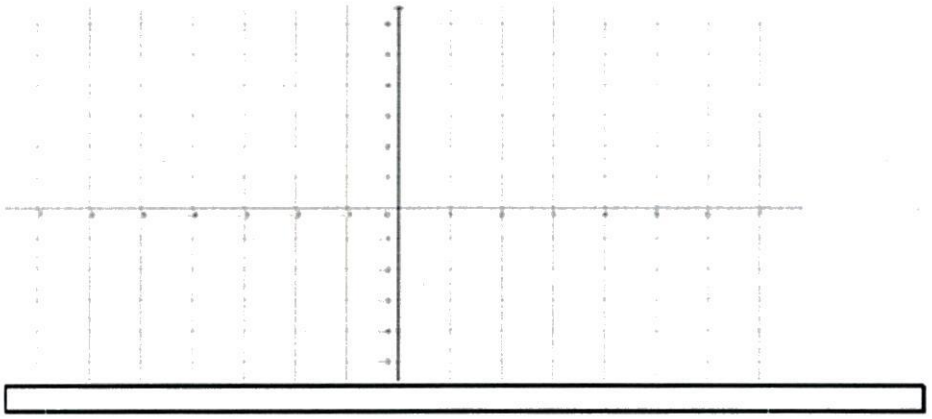


14. On a sunny day, a flagpole stands upright on the ground. The top of the flagpole is at point $A\ (4, 7)$, and the bottom of the flagpole is at point $B\ (4, 0)$. Determine the coordinates of the image of the flagpole when the ground acts as a mirror.

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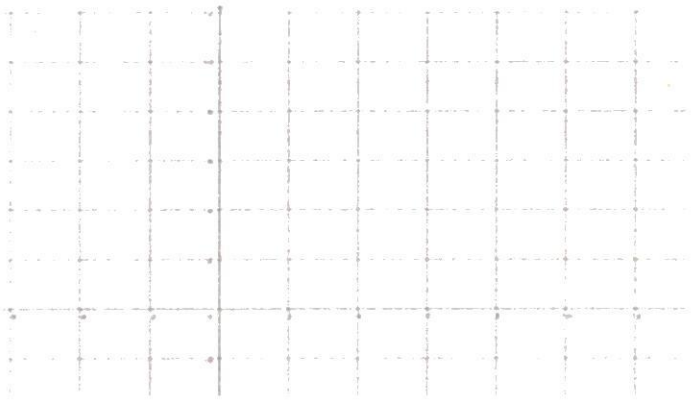


15. Given an object with vertices at $P(1,1), Q(1,4), R(4,4), and\ S(4,1)$ on a coordinate grid, and a transformation matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ plot the image of the object after the transformation. Describe the transformation.

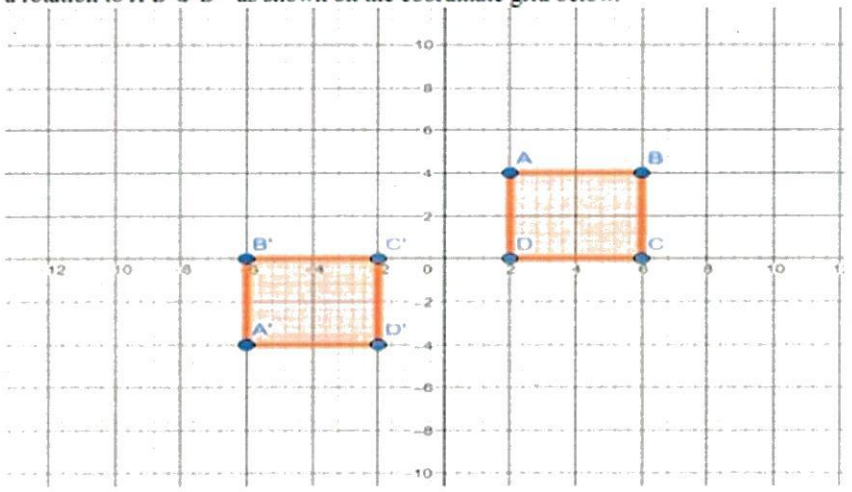


Level 4

16. A triangle has vertices at $(2,1), (4,1), and\ (2,3)$ on a coordinate grid. If it is dilated with a linear scale factor of 3. Calculate the area of the original triangle and the area of the dilated triangle. Show that the area scale factor is 9.

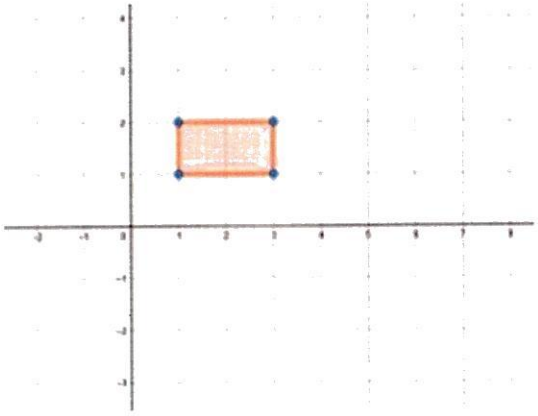


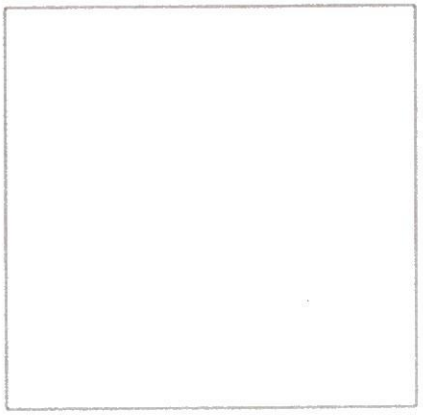
17. Find the angle of rotation and the center of rotation when $ABCD$ is transformed through a rotation to $A'B'C'D'$ as shown on the coordinate grid below.



18. Matrix $P = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$ transforms triangle ABC onto $A'B'C'$. While $A'B'C'$ is reflected throw the x-axis to give $A''B'''C''$. Write down the matrix that would transform $A''B'''C''$ directly back to ABC.

19. A student is designing a piece of artwork involving a house with a window that is initially smaller than required. The student wants to make the window larger such that the area ratio is 1:4. The window is represented by a rectangle with vertices at $A(1,1), B(1,2), C(3,1), and\ D(3,2)$. State the appropriate center of enlargement to ensure that the window remains in the same room and position. Write down the matrix of enlargement. Represent both the original window and the new window on a coordinate grid.





20. A biopharmaceutical company is designing an innovative delivery system for medication using drone technology. The drone needs to follow a precise path to reach multiple distribution points in a city, ensuring that the medication reaches all necessary locations efficiently. The drone's initial path is represented by a polygon with vertices $ABCD$ as shown on the coordinate grid below. The engineers want to transform this path to optimize the delivery route. The path needs to be rotated 90 degrees clockwise around the origin to avoid obstacles and improve delivery efficiency. After rotation, the path must be enlarged to cover a larger area, increasing the delivery range. To achieve this, the engineers decide to quadruple the area of the path. To ensure that the drone starts at a new central hub located at $(2,2)$, the entire path must be translated accordingly. Represent these transformations on the coordinate grip and write down the final coordinates of the vertices after these translations.



1.3 Pre-Test for Learners' Attitudes

We need your help with a short questionnaire about your feelings toward Transformation geometry. The test has 22 statements where you'll show your level of agreement or Disagreement.

Transformation geometry involves moving shapes in various ways, like sliding (translation), flipping (reflection), turning (rotation), or resizing (enlargement).

You tick ✔ under either 1 or 2 or 3 or 4 or 5 for each survey item to respectively mean:

1. Strongly Disagree with the statement
2. Disagree with the statement
3. Neutral about the statement or undecided about the statement
4. Agree with the statement
5. Strongly Agree with the statement

Your honest answers will help us better understand your thoughts on geometry and find effective ways to teach you.



School ID .....

Learner Personal number..... Gender: .....Age: .....

Learner Name .....

Table 1: Final Attitude Scale for Transformation Geometry (TGAS1)

S/N	Survey Items	1	2	3	4	5
Interest, Enjoyment & Confidence (Affective Component)						
1	I find transformation geometry interesting.					
2	Exploring geometric transformations attracts my attention.					
3	The concept of transformations in geometry excites me.					
4	I find joy in discovering the connections between shapes in transformation.					
5	I feel motivated to complete the transformation geometry classwork thoroughly.					
6	I am confident in my ability to understand transformation geometry concepts.					
7	I feel sure of myself when working on transformation geometry problems.					
Learning Engagement, Effort & Metacognition (Behavioral Component)						
8	I actively seek additional information about transformation geometry.					

S/N	Survey Items	1	2	3	4	5
9	I feel curious about the different types of transformations in geometry.					
10	I look forward to transformation geometry lessons.					
11	I seek opportunities to apply transformation geometry in real-world scenarios.					
12	Transformation geometry activities improved my learning experience.					
13	I take the initiative to discover transformational concepts on my own.					
14	I am confident that I can explain transformation geometry concepts to others.					
15	I feel capable of exploring new transformation geometry topics on my own.					
Relevance, Applications, & Importance (Cognitive Component)						
16	Understanding transformation geometry is important for my future.					
17	I can connect transformation geometry concepts to real-world situations.					
18	Transformation geometry is useful in fields outside of mathematics.					
19	I believe transformation geometry has applications beyond the classroom.					
20	Exploring transformation geometry enhanced my critical thinking skills.					
21	The knowledge of transformation geometry contributes to my overall education.					
22	I find value in understanding how transformations are used in various disciplines.					



Post-Test for Learners' Attitudes

We need your help with a short questionnaire about your feelings toward Transformation geometry. The test consists of 22 statements, where you'll indicate your level of agreement or Disagreement.

Transformation geometry involves moving shapes in various ways, like sliding (translation), flipping (reflection), turning (rotation), or resizing (enlargement).

You tick ✔ under either 1 or 2 or 3 or 4 or 5 for each survey item to respectively mean:



1. Strongly Disagree with the statement
2. Disagree with the statement
3. Neutral about the statement or undecided about the statement
4. Agree with the statement
5. Strongly Agree with the statement

Your honest answers will help us better understand your thoughts on geometry and be able to find effective ways of teaching you.

School ID .....

Learner Personal number..... Gender: .....Age: .....

Learner Name .....

Table 2: Final Attitude Scale for Transformation Geometry (TGAS2)

S/N	Survey Items	1	2	3	4	5
Interest, Enjoyment & Confidence (Affective Component)						
1	I think transformation geometry is motivating.					
2	I like exploring how shapes change in transformations.					
3	I'm not excited about the idea of transformations in geometry.*					
4	Finding connections between shapes in transformation makes me happy.					
5	I'm not excited about the idea of transformations in geometry.*					
6	Finding connections between shapes in transformation makes me happy.					
7	I feel good about myself when I work on transformation geometry problems.					
Learning Engagement, Effort & Metacognition (Behavioral Component)						
8	I enjoy looking for more information about transformation geometry.					
9	I wonder about the different types of transformations in geometry.					
10	I look forward to lessons about transformation geometry.					
11	I try to use what I learn in transformation geometry in real life.					
12	Doing different activities for transformation geometry makes learning more fun.					

1.4 Interview Guide

Introduction

Introduction

This interview is part of a study that aims to understand how students like you experience learning geometry, especially when using different teaching methods. Your thoughts and opinions are very important, they will help us understand what works well and what can be improved in the way geometry is taught.

There are no right or wrong answers. We just want to hear about your personal experience; how you felt during the lessons, what helped you learn, what was difficult, and any suggestions you might have. Feel free to speak freely and honestly. Everything you say will be kept confidential and used only for academic purposes.

If there's anything you don't understand, feel free to ask me to explain it again. You can also skip any question if you don't feel like answering it.

Question 1:
How was your experience learning geometry, and how did you get better at it over time?

Question 2:
How did the way you were taught affect how you learned geometry?

Question 3:
What parts of the teaching method were helpful or difficult for you, and how could the lessons be improved to help you learn better?

Question 4:
Did you have any unexpected difficulties while learning this transformation geometry?



Activity Work Plan

Table 24: Activity work Plan for the Study

S/N	ACTIVITY/MONTH	A	M	J	J	A	S	O	N	D	J	F	M	A
1	Proposal presentation and ethical clearance													
2	Mobilizing schools and teachers													
3	Piloting the tools													
4	Training the teachers/research assistants													
5	Conducting Pretest (GAS1 and VGAT1)													
6	Marking and Data entry for Pre-test													
7	Implementing the intervention													
8	Conducting the Posttest (GAS2 and VGAT2													
9	Marking and Data entry for Post-test													
10	Conducting follow-up interviews													
11	Conduct data cleaning													
12	Conduct preliminary Data Analysis and more literature review.													
13	Write and publish four papers, then submit them to the relevant journals.													
14	Conduct a teachers' workshop to disseminate research results													
15	Conference presentations													
16	Compile the Ph.D. thesis and submit it for examination.													

1.5 Sample Lesson Plan Based on Van Hiele Phased Instruction

Table 25: Sample Preliminary information for lesson plans

School:		Date:	
Subject:	Mathematics	Time:	
Teacher:		Duration:	
Class:		Number of Pupils:	
Term:		Boys:	
Theme:	Geometry	Girls:	

Topic:	**Transformation Geometry**
Competency:	Understanding and applying transformations to solve real-life problems.
Learning Outcome(s):	Determine the image given the object and transformation matrix on the coordinate grid.
Lesson Focus:	Knowledge, Understanding, Skills, Values, and Attitudes
Inputs for Teaching and Learning:	Paper, Pencil, Graph Paper, Computers, GeoGebra software, Ruler, Protractor
Pre-Requisite Knowledge:	Basic understanding of geometric shapes and coordinate geometry
References:	Mathematics Textbook, GeoGebra Manual, Online Geometry Resources

Table 26: Sample lesson Plan for CVHPI Class

Phase/Time	Teacher Activities: Observation, Conversation, Product	Learners' Activity: Discovery, Explanatory, Analysis and Application	Evidence of Achievement
Introduction 5 minutes	- Observe students' prior knowledge by asking questions about basic geometric shapes and coordinate geometry. - Converse with students to introduce the concept of transformations and their real-life applications.	- Discover and recall prior knowledge about geometric shapes and coordinate geometry.	- Students actively participate in recalling prior knowledge. - Students show interest in learning about transformations.
Information or Inquiry 10 minutes	- Introduce the basic concepts of applying a transformation matrix to an object to determine its image. - Provide learners with paper, pencil, and graph paper. - Ask them to plot a point (e.g., (2, 3)) and use simple transformation matrices to find the image of the point.	- Plot a point (e.g., (2, 3)) on graph paper. - Apply simple transformation matrices to find the image of the point.	- Students show engagement and curiosity about fundamental transformations. - Students can describe the effects of fundamental transformations.
Guided or Directed Orientation 10 minutes	- Provide specific tasks for students to apply different transformation matrices (reflection, rotation, and dilation/enlargement) to points. - Guide them to observe and record the coordinates before and after each transformation.	- Apply given transformation matrices to points and record the coordinates before and after transformations.	- Students accurately perform transformations and record their observations. - Students can identify patterns and relationships in transformations.
Explanation or Explication 10 minutes	- Explain how to identify and apply transformation matrices for each transformation. - Demonstrate the process using examples. - Encourage students to ask	- Listen to the explanation and follow along with examples. - Ask questions to clarify understanding. - Explain their	- Students demonstrate understanding by correctly identifying and applying transformation matrices.

Phase/Time	Teacher Activities: Observation, Conversation, Product	Learners' Activity: Discovery, Explanatory, Analysis and Application	Evidence of Achievement
	questions for clarification.	understanding using the correct terminology.	- Students use appropriate terminology to explain transformations.
Free Orientation 10 minutes	- Assign complex problems involving various transformation matrices. - Guide students to apply general transformation matrices to points to find the image.	- Independently solve tasks involving various transformation matrices. - Apply general transformation matrices to points.	- Students successfully apply transformation matrices to given tasks. - Students can explain the importance of the order of transformations.
Integration 10 minutes	- Summarize the learned concepts without introducing new material. - Provide a real-life application problem for review.	- Review how to apply reflection, rotation, dilation, and general transformation matrices to points. - Discuss real-life applications.	- Students present their solutions, demonstrating their ability to apply transformations in a real-life scenario. - Solutions show the correct use of transformation concepts and matrices. - Students can summarize the key concepts learned.
Conclusion 5 minutes	- Summarize key concepts of transformation geometry, focusing on combining transformations and their real-life applications. - Answer any remaining questions from students.	- Reflect on the lesson and ask questions if any. - Understand the practical applications of combining transformations.	- Students accurately summarize the lesson's key points. - Students show an understanding of the practical applications of combining transformations.

Teacher Self-Assessment:

- Reflect on the effectiveness of the lesson plan and student engagement.
- Assess whether the learning objectives were met.
- Identify any areas for improvement in teaching transformation geometry.
- Note students' feedback and adapt future lessons accordingly.

Table 27: Sample Lesson Plan for TVHPI Class

Phase/Time	Teacher Activities: Observation, Conversation, Product	Learners' Activity: Discovery, Explanatory, Analysis, and Application	Evidence of Achievement
Introduction 5 minutes	- Observe students' prior knowledge by asking questions about basic geometric shapes and coordinate geometry. - Converse with students to introduce the concept of transformations and their real-life applications.	- Discover and recall prior knowledge about geometric shapes and coordinate geometry.	- Students actively participate in recalling prior knowledge. - Students show interest in learning about transformations.
Information or Inquiry 10 minutes	- Introduce the basic concepts of applying a transformation matrix to an object to determine its image through dynamic visualizations. - Use GeoGebra to plot a point (e.g., (2, 3)) and apply simple transformation matrices to find the image of the point.	- Use GeoGebra to plot a point (e.g., (2, 3)) and apply simple transformation matrices to find the image of the point.	- Students show engagement and curiosity about basic transformations. - Students can describe the effects of basic transformations.
Guided or Directed Orientation 15 minutes	- Provide specific tasks for students to apply different transformation matrices (reflection, rotation, and dilation/enlargement) to points using GeoGebra. - Guide them to observe and record the coordinates before and after each transformation.	- Use GeoGebra tools to apply given transformation matrices to points and record the coordinates before and after transformations.	- Students accurately perform transformations and record their observations. - Students can identify patterns and relationships in transformations.
Explanation or Explication 15 minutes	- Explain how to identify and apply transformation matrices for each transformation using GeoGebra.	- Listen to the explanation and follow along with examples in GeoGebra.	- Students demonstrate understanding by correctly identifying and applying transformation matrices in GeoGebra.

Phase/Time	Teacher Activities: Observation, Conversation, Product	Learners' Activity: Discovery, Explanatory, Analysis, and Application	Evidence of Achievement
	- Demonstrate the process using examples in GeoGebra. - Encourage students to ask questions for clarification.	- Ask questions to clarify understanding. - Explain their understanding using the correct terminology.	- Students use appropriate terminology to explain transformations.
: Free Orientation 20 minutes	- Assign complex problems involving various transformation matrices. - Guide students to apply general transformation matrices to points to find the image using GeoGebra.	- Use GeoGebra to independently solve tasks involving various transformation matrices. - Apply general transformation matrices to points in GeoGebra.	- Students successfully apply transformation matrices to given tasks using GeoGebra. - Students can explain the importance of the order of transformations.
Integration 10 minutes	- Summarize the learned concepts using GeoGebra without introducing new material. - Provide a real-life application problem for review in GeoGebra.	- Review how to apply reflection, rotation, dilation, and general transformation matrices to points using GeoGebra. - Discuss real-life applications using GeoGebra.	- Students present their solutions, demonstrating their ability to apply transformations in a real-life scenario using GeoGebra. - Solutions show the correct use of transformation concepts and matrices in GeoGebra. - Students can summarize the key concepts learned.
Conclusion 5 minutes	- Summarize key concepts of transformation geometry, focusing on combining transformations and their real-life applications using GeoGebra. - Answer any remaining questions from students.	- Reflect on the lesson and ask questions if any. - Understand the practical applications of combining transformations	- Students accurately summarize the lesson's key points. - Students show an understanding of the practical applications of combining transformations.

Phase/Time	Teacher Activities: Observation, Conversation, Product	Learners' Activity: Discovery, Explanatory, Analysis, and Application	Evidence of Achievement
		using GeoGebra.	

Teacher Self-Assessment:

- Reflect on the effectiveness of the lesson plan and student engagement.
- Assess whether the learning objectives were met.
- Identify any areas for improvement in teaching transformation geometry.
- Note students' feedback and adapt future lessons accordingly.

1.6 Activities under each van Hiele level (a case of transformation geometry).

The activities under each Van Hiele level in transformation geometry highlight the progression of geometric reasoning skills from basic visualization and recognition (Level 1) to advanced formal proofs and axiomatic systems (Level 5). These activities provide insights into the depth and complexity of geometric understanding required at each stage of cognitive development, informing tailored instructional strategies and curriculum design, as illustrated in Figure 25.

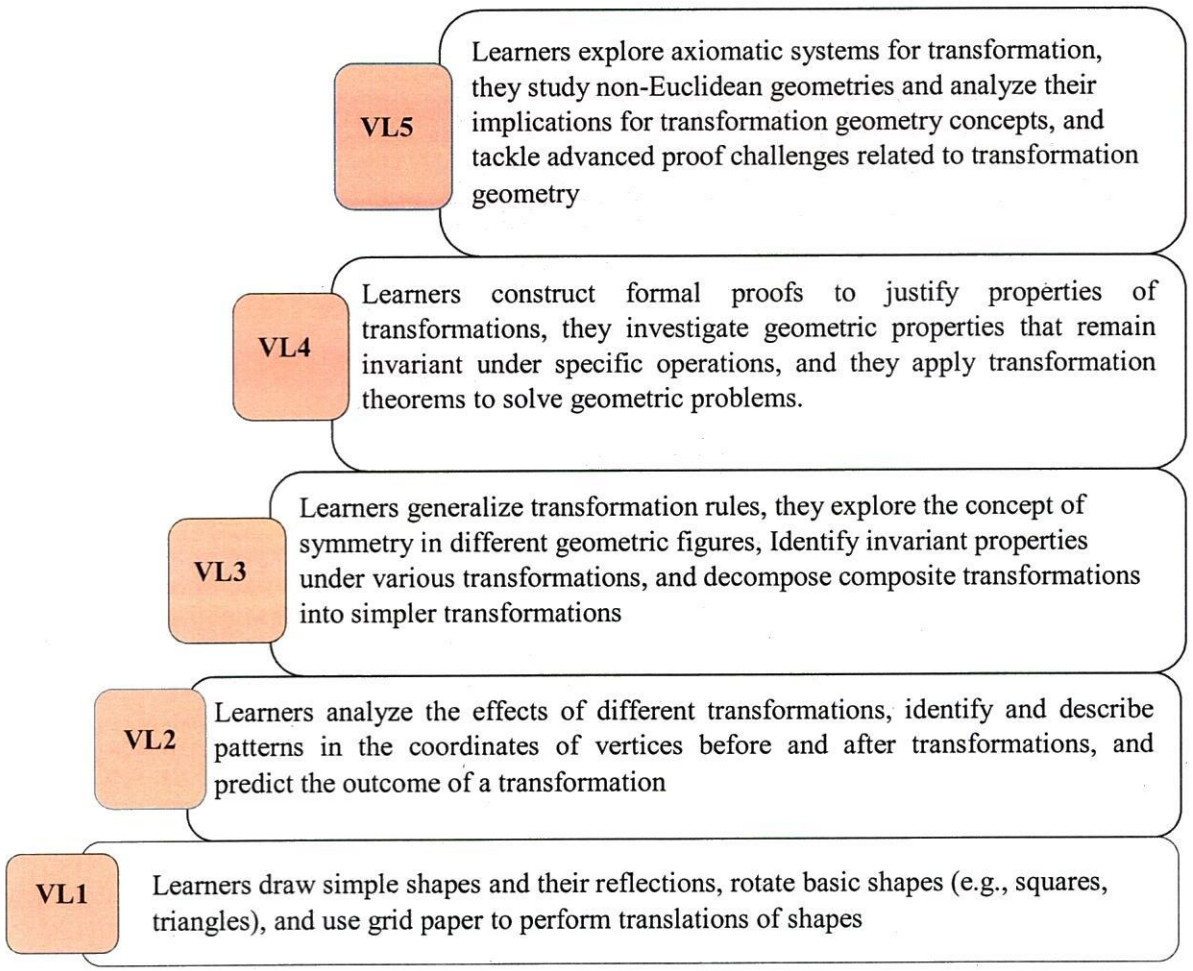


Figure 25: Activities under Each Van Hiele Level (a case of Transformation Geometry).

1.7 Proposal Approvals.

Appendix B: Proposal Approval Form

MBARARA UNIVERSITY OF SCIENCE AND TECHNOLOGY

PROPOSAL APPROVAL FORM

To: **The Director of Research and Graduate Training**

Name: **ISSA NDUNGO**

Registration #: **2022/Ph.D/044/PS** Student #: **2200601323**

Programme: **Ph.D. IN CURRICULUM STUDIES (MATHEMATICS EDUCATION)**

Department: **EDUCATIONAL FOUNDATIONS AND PSYCHOLOGY**

Faculty/Institute: **SCIENCE**

Academic Year: **TWO**

Semester: **ONE**

This is to certify that the proposal titled:
The Interplay among Instructional Strategies, Learners' Geometry Attitudes, and Van Hieles Levels of Geometry Reasoning

was approved for a study leading to the award of
Ph.D. IN CURRICULUM STUDIES (MATHEMATICS EDUCATION)

of Mbarara University of Science and Technology

Signature/Stamp: Date: 15th April 2024

HEAD OF DEPARTMENT

MBARARA UNIVERSITY P.O. BOX 1410, MBARARA 15 APR 2024 DEPARTMENT OF EDUCATIONAL FOUNDATIONS & PSYCHOLOGY

cc: Faculty Dean

1.10 REC Approval

MBARARA UNIVERSITY OF SCIENCE AND TECHNOLOGY

Mbarara University of Science & Technology
P. O. Box 1410, Mbarara
RESEARCH ETHICS COMMITTEE
03 JUL 2024
APPROVED
VALID UNTIL DATE SHOWN ABOVE

03/07/2024

To: Issa Ndungo

Mbarara University of Science and Technology
+256776428589

Type: Initial Review

Re: MUST-2024-1519: The Interplay among Instructional Strategies, Learners Geometry Attitudes, and Van Hiele Levels of Geometric Reasoning: A Case Study of Selected Secondary Schools in Mid-Western Uganda.

I am pleased to inform you that the MUST Research Ethics Committee, through expedited review held on **06/06/2024** approved the above referenced study.
Approval of the research is for the period of **03/07/2024** to **03/07/2025.**

As Principal Investigator of the research, you are responsible for fulfilling the following requirements of approval:

1. All co-investigators must be kept informed of the status of the research.
2. Changes, amendments, and addenda to the protocol or the consent form must be submitted to the REC for re-review and approval **prior** to the activation of the changes.
3. Reports of unanticipated problems involving risks to participants or any new information which could change the risk benefit: ratio must be submitted to the REC.
4. Only approved consent forms are to be used in the enrollment of participants. All consent forms signed by participants and/or witnesses should be retained on file. The REC may conduct audits of all study records, and consent documentation may be part of such audits.
5. Continuing review application must be submitted to the REC **eight weeks** prior to the expiration date of **03/07/2025** in order to continue the study beyond the approved period. Failure to submit a continuing review application in a timely fashion may result in suspension or termination of the study.
6. The REC application number assigned to the research should be cited in any correspondence with the REC of record.
7. You are required to register the research protocol with the Uganda National Council for Science and Technology (UNCST) for final clearance to undertake the study in Uganda.

The following is the list of all documents approved in this application by MUST Research Ethics Committee:

No.	Document Title	Language	Version Number	Version Date
1	stakeholder engagement plan	English	1	2024-06-11
2	Training Protocol	English	1	2024-06-11
3	Protocol	English	clear	2024-06-11
4	Assent form if applicable to your study	English	1	2024-06-10
5	Informed Consent forms	English	2	2024-05-23
6	Data collection tools	English	1	2024-04-11

Yours Sincerely

Dr. Paul Alele
For: MUST Research Ethics Committee



1.11. UNCST Approval



Uganda National Council for Science and Technology

(Established by Act of Parliament of the Republic of Uganda)

Our Ref: SS2857ES **6 August 2024**

Issa Ndungo
Mbarara University of Science and Technology
Mbarara

Re: Research Approval: **The Interplay among Instructional Strategies, Learners Geometry Attitudes, and Van Hiele Levels of Geometric Reasoning: A Case Study of Selected Secondary Schools in Mid-Western Uganda.**

I am pleased to inform you that on **06/08/2024**, the Uganda National Council for Science and Technology (UNCST) approved the above referenced research project. The Approval of the research project is for the period of **06/08/2024** to **06/08/2025**.

Your research registration number with the UNCST is **SS2857ES**. Please, cite this number in all your future correspondences with UNCST in respect of the above research project. As the Principal Investigator of the research project, you are responsible for fulfilling the following requirements of approval:

1. Keeping all co-investigators informed of the status of the research.
2. Submitting all changes, amendments, and addenda to the research protocol or the consent form (where applicable) to the designated Research Ethics Committee (REC) or Lead Agency for re-review and approval **prior** to the activation of the changes. UNCST must be notified of the approved changes within five working days.
3. For clinical trials, all serious adverse events must be reported promptly to the designated local REC for review with copies to the National Drug Authority and a notification to the UNCST.
4. Unanticipated problems involving risks to research participants or other must be reported promptly to the UNCST. New information that becomes available which could change the risk/benefit ratio must be submitted promptly for UNCST notification after review by the REC.
5. Only approved study procedures are to be implemented. The UNCST may conduct impromptu audits of all study records.
6. An annual progress report and approval letter of continuation from the REC must be submitted electronically to UNCST. Failure to do so may result in termination of the research project.

Please note that this approval includes all study related tools submitted as part of the application as shown below:

No.	Document Title	Language	Version Number	Version Date
1	Informed Consent forms	English	2	23 May 2024
2	Data collection tools	English	1	11 April 2024
3	Project Proposal	English	CLEAR	
4	Approval Letter	English		
5	Administrative Clearance	English		

Yours sincerely,

Hellen Opolot
For: Executive Secretary
UGANDA NATIONAL COUNCIL FOR SCIENCE AND TECHNOLOGY

LOCATION/CORRESPONDENCE

Plot 6 Kimera Road, Ntinda
P.O. Box 6884
KAMPALA, UGANDA

COMMUNICATION

TEL: (256) 414 705500
FAX: (256) 414-234579
EMAIL: info@uncst.go.ug
WEBSITE: http://www.uncst.go.ug

1.12. Approved Consent form

MBARARA UNIVERSITY OF SCIENCE AND TECHNOLOGY

RESEARCH ETHICS COMMITTEE

P.O. Box 1410 Mbarara, Tel: +256-48-543-3795, Fax: +256-48-542-0782

E-mail: irc@must.ac.ug, mustirb@gmail.com

INFORMED CONSENT FORM

This document outlines the research study and expectations for potential participants. It should be written in layman terms and typed on MUST-REC letterhead

Instructions

1. The wording of this document should be directed to the potential participant, not MUST-REC
2. If a technical term must be used, then define it the first time it is used and any acronyms or abbreviations used should be spelled out the first time they are used.
3. All the sections of this document must be completed without any editing or deletions
4. Please use a typing font that is easily distinguishable from the questions of this form. Preferably the font size should be 12.

Study title – This should be the same as on all other documents related to the study.

The Interplay among Instructional Strategies, Learners' Geometry Attitudes, and Van Hiele Levels of Geometric Reasoning: A Case Study of Selected Secondary Schools of Mid-Western Uganda

Principal Investigator(s)

Issa Ndungo

Introduction

What you should know about this study:

1. You are being asked to join a research study.
2. This consent form explains the research study and your part in the study.
3. Please read it carefully and take as much time as you need.
4. You are a volunteer. You can choose not to take part and if you join, you may quit at any time. There will be no penalty if you decide to quit the study

Leave blank for REC office only:

MUST-REC Stamp:

RESEARCH ETHICS COMMITTEE MBARARA UNIVERSITY OF SCIENCE AND TECHNOLOGY P.O BOX 1410 MBARARA

REC OFFICE USE ONLY:
APPROVAL DATE 03/07/2024
APPROVED CONSENT IRB VERSION NUMBER 2024-06-11
PI NAME Issa Ndungo
IRB NO MUST-2024-1919

Brief background to the study

The study investigates geometry education in Midwestern Uganda, addressing widespread challenges in learners' understanding and attitudes toward geometry. Despite its foundational importance, geometry education faces significant obstacles globally, including low achievement and negative attitudes toward the subject. In Uganda, traditional instructional practices and curriculum transitions compound these challenges, particularly in the context of the Competency-Based Curriculum (CBC). The study focuses on lower secondary schools in Midwestern Uganda, aiming to assess the effectiveness of instructional strategies, including Van Hiele phased instruction and technology-enhanced Van Hiele phased instruction, in enhancing learners' geometry proficiency within the CBC framework. The research adopts a comprehensive theoretical framework integrating constructivist learning theory and the Van Hiele theory, aiming to explore the interplay between instructional strategies, learners' geometry attitudes, and Van Hiele's levels of geometric reasoning. By providing empirical insights, the study seeks to inform evidence-based geometry education strategies tailored to the region's educational system, ultimately contributing to improved educational outcomes and learner success.

Purpose of the research project

The study involves research conducted in selected lower secondary schools of Midwestern Uganda, with 579 participants, including both teachers and students. The purpose of the research procedure is to assess the effectiveness of instructional strategies, including Van Hiele phased instruction and technology-enhanced Van Hiele phased instruction, in enhancing learners' geometry proficiency within the Competency-Based Curriculum (CBC) framework. The intervention will last for six weeks, with teachers engaging in training sessions, classroom observations, and data collection on learners' attitudes and geometric reasoning levels

Why you are being asked to participate?

You were selected to participate in the study based on your affiliation with the selected lower secondary schools in Midwestern Uganda. This selection process ensures that individuals directly involved in the educational context under investigation are included, allowing for a comprehensive assessment of the effectiveness of instructional strategies.

Procedures

It's critical that you comprehend the study's processes before starting this research adventure with us. A two-day training session on Van Hiele phased instruction and technology-enhanced Van Hiele phased instruction will be held for a chosen group of instructors to kick off the project. After that, for a period of six weeks, geometry classes utilizing these instructional methodologies will be given. To guarantee fidelity in execution, classroom activities will be continuously observed and monitored

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APPROVAL DATE 03/07/2024
APPROVED CONSENT IRB VERSION NUMBER 2024-06-17
PI NAME Issa Ndungo
IRB NO. MUREC-2024-1519

during the intervention. Data on learners' attitudes toward geometry and their Van Hiele levels of geometric reasoning will be gathered at the start and finish of the instruction. After the intervention, a limited group of students will also be interviewed to obtain additional insights.

Risks or discomforts

Even though the study's main objectives are data gathering and instructional interventions, being watched while participating in class activities and answering geometry competency tests could cause some psychological distress. Strict confidentiality measures will be put in place to safeguard your privacy and anonymity to guarantee your safety and well-being. You also have the freedom to leave the study at any moment and without penalty. It's important to realize that participation is completely voluntary and that you always have the option to withdraw if something makes you uncomfortable.

Benefits

Your involvement in this study offers unique benefits tailored to both teachers and learners. For teachers, active participation in the training sessions and implementation of instructional strategies can enhance their pedagogical skills and repertoire, equipping them with innovative teaching methods that promote student engagement and comprehension of geometry concepts. Additionally, for learners, engaging in educational interventions and assessments focused on geometry proficiency can lead to improved understanding and mastery of geometric principles, ultimately enhancing academic performance and problem-solving skills.

Incentives or rewards for participating

We much appreciate your participation in this research. As an incentive, we will reimburse teachers for the cost of their transportation to cover any applicable travel expenses. Every learner who takes part in the study will also receive a mathematical set or a calculator

Protecting data confidentiality

This study places a high priority on confidentiality. All data and records gathered from you will be treated with the utmost confidentiality. Any papers or publications that come out of this research will not reveal any personally identifiable information.

Protecting subject privacy during data collection

Your privacy will be strictly maintained during the data collection procedure. The acquired data will be securely stored in password-protected electronic databases, with access restricted to authorized research professionals only. All digital information will be secured with encryption to avoid unwanted access. Furthermore, you will be guaranteed confidentiality throughout interviews or observations, and no personally identifiable information will be disclosed. To further protect privacy, your identities will be anonymized in any study-related reports or publications

Leave blank for REC office only:

MUST HAVE Stamp



REC OFFICE USE ONLY:
APPROVAL DATE: 03/07/2024
APPROVED CONSENT IRB VERSION NUMBER: 2024-06-11
PI NAME: Issa Ndungo
IRB NO: MUST-2024-1519

Right to refuse or withdraw

You are free to decline participation in this study at any time without incurring any penalties or losing any rewards to which you would otherwise be entitled.

What happens if you leave the study?

You may opt out of the study at any moment without facing any punishments, and opting not to participate will also carry no disadvantages.

Who do I ask/call if I have questions or a problem?

Assoc. Prof. Paul Alele	Issa Ndungo
Chairman, MUST-REC	Principal Investigator
P.O Box 1410 Mbarara	P o Box 837, Fort Portal
Tel: 0485433795/0773573372	Tel: 0776428589

What does your signature or thumbprint on this consent form mean?

Your signature on this form means

- You have been informed about this study's purpose, procedures, possible benefits and risks
- You have been given the chance to ask questions before you sign
- You have voluntarily agreed to be in this study

Name of adult participant	Signature/Thumbprint of participant Parent/Guardian/Next of Kin	Date
Name of person obtaining consent	Signature	Date
Print Name of witness	Signature or thumbprint or mark	Date

Leave blank for REC otherwise

MUST-REC Stamp



REC OFFICE USE ONLY:
APPROVAL DATE: 03/07/2024
APPROVED CONSENT IRB VERSION NUMBER: 2024-06-11
PI NAME: Issa Ndungo
IRB NO: MUST-2024-1519

1.13. Assent form

MBARARA UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF EDUCATION FOUNDATIONS AND PSYCHOLOGY

P. O. BOX 1410,

MBARARA- UGANDA

Learner Assent Form for Participation in a Geometry Study

Introduction:

Hello! My name is Issa Ndungo, and I am conducting a study to understand how different ways of teaching geometry can affect how you feel about and understand it. You are invited to join this study because you are in Senior Three (S.3), and we believe your participation can help us learn a lot.

Purpose of the Study:

This study aims to;

1. Compare how different teaching methods affect your attitude towards geometry.
2. See how your attitude towards geometry affects your understanding and progression in geometric reasoning.
3. Compare how different teaching methods affect your progression in understanding geometry.
4. Determine if your attitude towards geometry influences the relationship between teaching methods and your progression in geometric reasoning.

What Will Happen During the Study?

1. You will be taught transformation geometry using either traditional or technology-enhanced Van Hiele-phased instructional strategies.
2. You will take tests before and after the teaching sessions to measure your understanding of geometry.
3. You will complete surveys about your attitudes towards geometry.
4. Some of you might be interviewed and observed during the lessons.

You're Rights

1. Your participation is entirely voluntary. This means you do not have to take part if you do not want to.
2. If you decide to take part, you can stop at any time without giving a reason.
3. Your grades and standing in school will not be affected by your decision to participate or not participate.

RESEARCH ETHICS COMMITTEE

Benefits of Participating

1. You will help us understand how to improve the way geometry is taught.
2. You may find that your understanding and attitude towards geometry improve.

Risks:

1. There are no known risks associated with this study. However, if you feel uncomfortable at any time, you can choose to stop participating.

Confidentiality:

1. Your responses and test scores will be kept confidential. Your name will not be used in any reports or publications.
2. The data collected will be stored securely, and only the research team will have access to it.

Questions:

If you have any questions about the study, you can ask me now or later. If you are unsure, you can also talk to your parents or teachers.

Assent:

If you want to participate in this study, please sign your name below. This means you understand what the study is about, and you agree to take part.

Thank you for considering being part of this study!

Participant's Name (minor) .....

Participant's Signature (minor) .....

Date .....

Researcher's Name .....

Signature .....

Date .....



1.14. Administrative Approval

MBARARA UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE

DEPARTMENT OF EDUCATIONAL FOUNDATIONS AND PSYCHOLOGY

P.O. Box 1410 Mbarara, Tel: +256-48-543-3795, Fax: +256-48-542-0782

E-mail: issandungo1@gmail.com/ndungoissa@must.ac.ug

24th May 2024

The Headteacher

Munkunyu Secondary School

Kasese District

Received
28/5/2024

HEADTEACHER
MUNKUNYU SEC. SCHOOL
P.O.BOX 539, KASESE

Dear Sir,

Request for Permission to Conduct Ph.D Research Study in Your School

My name is Issa Ndungo and I am a Ph.D student at Mbarara University of Science and Technology. I am writing to seek permission to conduct the Ph.D research study in your school.

The purpose of this Ph.D study is to investigate the effectiveness of instructional strategies, particularly the implementation of Van Hiele phased instruction and the utilization of GeoGebra, in teaching Transformation Geometry within the Competency-Based Curriculum (CBC) framework.

During the study, selected mathematics teachers will be trained and monitored during the teaching of transformation geometry in the S.3 class. Data on learners' geometry attitudes and Van Hiele levels will be collected at the intervention's start and end. Activities are scheduled from June to October 2024 to align with school programs. Data confidentiality is ensured; participation is voluntary and identities will be kept anonymous in research reports and publications.

Your support and cooperation in facilitating the research activities within this school is valued.

Yours sincerely

.....

Issa Ndungo

Ph.D Student- 2022/PhD/044/PS

+256776428589/705686509

PERMISSION DECISION

Permission granted.

28/5/2024

HEADTEACHER
MUNKUNYU SEC. SCHOOL
P.O.BOX 539, KASESE

(Authorized signature and Stamp)

MBARARA UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE

DEPARTMENT OF EDUCATIONAL FOUNDATIONS AND PSYCHOLOGY

P.O. Box 1410 Mbarara, Tel: +256-48-543-3795, Fax: +256-48-542 0782

E-mail: issandungo1@gmail.com/ndungoissa@mmu.ac.ug

24th May 2024

The District Education Officer, Kasese District

Received and permission granted to conduct research in secondary schools in Kasese District

28 MAY 2024

DISTRICT INSPECTOR OF SCHOOLS KASESE

Dear Sir,

Request for Permission to Conduct Ph.D Research Study in Sec. Schools of Kasese District.

My name is Issa Ndungo, and I am a Ph.D. student at Mbarara University of Science and Technology. I'm writing to request authorization to do the Ph.D. research study in a couple of secondary schools in Kasese District. The purpose of this Ph.D study is to investigate the effectiveness of instructional strategies, particularly the implementation of Van Hiele phased instruction and the utilization of GeoGebra, in teaching Transformation Geometry within the Competency-Based Curriculum (CBC) framework.

A subset of teachers will get training and ongoing observation during the study while they instruct the S.3 class on transformation geometry. At the beginning and end of the intervention, information on the geometry attitudes and Van Hiele levels of the learners will be gathered. June through October of 2024 is when the activities are intended to coincide with schools' programs. Participation is voluntary, data confidentiality is guaranteed, and identities in research reports and publications will remain anonymous.

Your support and cooperation in facilitating the research activities within the district is valued.

Yours sincerely

.....

Issa Ndungo
Ph.D Student- 2022/PhD/044/PS
+256776428589/705686509

PERMISSION DECISION

MBARARA UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE

DEPARTMENT OF EDUCATIONAL FOUNDATIONS AND PSYCHOLOGY

P.O. Box 1410 Mbarara, Tel: +256-48-543-3795, Fax: +256-48-542-0782

E-mail: issandungo1@gmail.com/ndungoissa@mmu.ac.ug

24th May 2024

The Headteacher

Kyarumba Islamic Secondary School

Kasese District

Dear Sir,

Request for Permission to Conduct Ph.D Research Study in Your School

My name is Issa Ndungo and I am a Ph.D student at Mbarara University of Science and Technology. I am writing to seek permission to conduct the Ph.D research study in your school.

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Your support and cooperation in facilitating the research activities within this school is valued.

Yours sincerely

.....

Issa Ndungo

Ph.D Student- 2022/PhD/044/PS

+256776428589/705686509

PERMISSION DECISION

10 JUN 2024 Permission Granted

MBARARA UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE

DEPARTMENT OF EDUCATIONAL FOUNDATIONS AND PSYCHOLOGY

P.O. Box [illegible] Mbarara, Tel: +256-48-543-3795, Fax: +256-48-542-0782

E-mail: [illegible]@mmu.ac.ug

24th May 2024

The Headteacher

Mutanywana Secondary School

Kasese District

HEADTEACHER
17 MAY 2024
MUTANYWANA SEC. SCH.
P. O. BOX 79, KASESE

Dear Sir,

Request for Permission to Conduct Ph.D Research Study in Your School

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Your support and cooperation in facilitating the research activities within this school is valued.

Yours sincerely

Issa Ndungo
Ph.D Student- 2022/PhD/044/PS
+256776428589 [illegible]

PERMISSION DECISION

HEADTEACHER
17 MAY 2024
MUTANYWANA SEC. SCH.
P. O. BOX 79, KASESE

MBARARA UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE

DEPARTMENT OF EDUCATIONAL FOUNDATIONS AND PSYCHOLOGY

P.O. Box 1410 Mbarara, Tel: +256-48-543-3795, Fax: +256-48-542-0782

E-mail: issandungo1@gmail.com/ndungoissa@mmu.ac.ug

24th May 2024

The Headteacher

Fort Portal Secondary School

Fort Portal City

Dear Sir,

Request for Permission to Conduct Ph.D Research Study in Your School

My name is Issa Ndungo and I am a Ph.D student at Mbarara University of Science and Technology. I am writing to seek permission to conduct the Ph.D research study in your school.

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During the study, selected mathematics teachers will be trained and monitored during the teaching of transformation geometry in the S.3 class. Data on learners' geometry attitudes and Van Hiele levels will be collected at the intervention's start and end. Activities are scheduled from June to October 2024 to align with school programs. Data confidentiality is ensured; participation is voluntary and identities will be kept anonymous in research reports and publications.

Your support and cooperation in facilitating the research activities within this school is valued.

Yours sincerely

.....

Issa Ndungo

Ph.D Student- 2022/PhD/044/PS

+256776428589/705686509

PERMISSION DECISION

[Author and Signature and Stamp]

FORT PORTAL SENIOR SECONDARY SCHOOL
04 JUN 2024
P. O. BOX 457, FORT-PORTAL
HEADTEACHER

MBARARA UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE

DEPARTMENT OF EDUCATIONAL FOUNDATIONS AND PSYCHOLOGY

P.O. Box 1410 Mbarara, Tel: +256-48-543-3795, Fax: +256-48-542-0782

E-mail: issandungo1@gmail.com/ndungoissa@mmu.ac.ug;

24th May 2024

The City Education Officer, For-Portal City

Dear Sir,

Request for Permission to Conduct Ph.D Research Study in Sec. Schools of Fort Portal City

My name is Issa Ndungo, and I am a Ph.D. student at Mbarara University of Science and Technology. I'm writing to request authorization to do the Ph.D. research study in a couple of secondary schools in Fort Portal City. The purpose of this Ph.D study is to investigate the effectiveness of instructional strategies, particularly the implementation of Van Hiele phased instruction and the utilization of GeoGebra, in teaching Transformation Geometry within the Competency-Based Curriculum (CBC) framework.

A subset of teachers will get training and ongoing observation during the study while they instruct the S.3 class on transformation geometry. At the beginning and end of the intervention, information on the geometry attitudes and Van Hiele levels of the learners will be gathered. June through October of 2024 is when the activities are intended to coincide with schools' programs. Participation is voluntary, data confidentiality is guaranteed, and identities in research reports and publications will remain anonymous.

Your support and cooperation in facilitating the research activities within the City is valued.

Yours sincerely

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Issa Ndungo
Ph.D Student- 2022/PhD/044/PS
+256776428589/705686509

PERMISSION DECISION

CITY FORT PORTAL
Permission granted
1-10-2024

MBARARA UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF SCIENCE

DEPARTMENT OF EDUCATIONAL FOUNDATIONS AND PSYCHOLOGY

P.O. Box 1410 Mbarara, Tel: +256-48-543-3795, Fax: +256-48-542-0782

E-mail: issandungo16@gmail.com/ndungoissa@mmu.ac.ug

24th May 2024

The Headteacher

Mpanga secondary School

Fort Portal City

Dear Sir,

Request for Permission to Conduct Ph.D Research Study in Your School

My name is Issa Ndungo and I am a Ph.D student at Mbarara University of Science and Technology. I am writing to seek permission to conduct the Ph.D research study in your school.

The purpose of this Ph.D study is to investigate the effectiveness of instructional strategies, particularly the implementation of Van Hiele phased instruction and the utilization of GeoGebra, in teaching Transformation Geometry within the Competency-Based Curriculum (CBC) framework.

During the study, selected mathematics teachers will be trained and monitored during the teaching of transformation geometry in the S.3 class. Data on learners' geometry attitudes and Van Hiele levels will be collected at the intervention's start and end. Activities are scheduled from June to October 2024 to align with school programs. Data confidentiality is ensured; participation is voluntary and identities will be kept anonymous in research reports and publications.

Your support and cooperation in facilitating the research activities within this school is valued.

Yours sincerely

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Issa Ndungo

Ph.D Student- 2022/PhD/044/PS

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PERMISSION DECISION

Request granted

HEADTEACHER MPANGA SEC. SCHOOL FORT PORTAL