

Comparing technology-enhanced and conventional Van Hiele Phased Instruction in advancing learners through Van Hiele levels in transformation geometry

Issa Ndungo, Sudi Balimuttajjo , Edwin Akugizibwe & Charles Magoba Muwonge

To cite this article: Issa Ndungo, Sudi Balimuttajjo , Edwin Akugizibwe & Charles Magoba Muwonge (2026) Comparing technology-enhanced and conventional Van Hiele Phased Instruction in advancing learners through Van Hiele levels in transformation geometry, Cogent Education, 13:1, 2716983, DOI: [10.1080/2331186X.2026.2716983](https://doi.org/10.1080/2331186X.2026.2716983)

To link to this article: <https://doi.org/10.1080/2331186X.2026.2716983>



© 2026 The Author(s). Published by Informa UK Limited, trading as Taylor & Francis Group



Published online: 12 Aug 2026.



Submit your article to this journal [↗](#)



Article views: 151



View related articles [↗](#)



View Crossmark data [↗](#)

Comparing technology-enhanced and conventional Van Hiele Phased Instruction in advancing learners through Van Hiele levels in transformation geometry

Issa Ndungo^{a,b} , Sudi Balimuttajjo^b, Edwin Akugizibwe^a and Charles Magoba Muwonge^{b,c}

^aMountains of the Moon University, Fort-Portal, Uganda; ^bMbarara University of Science and Technology, Mbarara City, Uganda; ^cUganda Martyrs University, Kampala, Uganda

ABSTRACT

Developing learners' geometric reasoning remains a persistent challenge in secondary mathematics education, particularly in transformation geometry, where many learners struggle to progress through the hierarchical Van Hiele levels of geometric thinking. Although technology-enhanced instruction has been associated with improved mathematics learning, limited evidence exists on how integrating GeoGebra within the phases of Van Hiele instruction influences learners' progression through these levels. This study compared Technology-Enhanced Van Hiele Phased Instruction (TVHPI) with Conventional Van Hiele Phased Instruction (CVHPI) in promoting learners' progression through the Van Hiele levels in transformation geometry. Guided by Van Hiele theory and constructivist learning principles, a quasi-experimental design involving 483 Ugandan secondary school learners (TVHPI = 238; CVHPI = 245) was employed. Learners' progression was assessed using the Van Hiele Geometry Achievement Test and analysed using Wilcoxon signed-rank tests, ordinal logistic regression, and Difference-in-Differences analysis. Both instructional approaches significantly improved learners' geometric reasoning; however, learners receiving TVHPI demonstrated greater progression through the Van Hiele levels than those receiving CVHPI. The findings suggest that integrating GeoGebra within the phases of Van Hiele instruction is associated with higher levels of geometric reasoning and provides empirical support for technology-enhanced, competency-based geometry instruction that promotes deeper conceptual understanding.

ARTICLE HISTORY

Received 21 November 2025
Revised 27 July 2026
Accepted 6 August 2026

KEYWORDS

Technology-enhanced instruction; Geometric reasoning; Transformation geometry; Van Hiele levels; GeoGebra

SUBJECTS

Mathematics Education;
Pedagogy; Curriculum;
Geometry Education

Introduction

Background

Geometry plays a fundamental role in developing learners' spatial reasoning, visualization, and problem-solving abilities, which are essential for success in mathematics and related disciplines (Asli & Zsoldos-Marchis, 2021). Within the secondary mathematics curriculum, transformation geometry provides an important link between geometric and algebraic reasoning through concepts such as translation, reflection, rotation, and dilation, making it particularly important for the development of higher-order geometric reasoning (National Curriculum Development Centre [NCDC], 2019a, 2019b).

Despite its importance, transformation geometry remains one of the most challenging areas of secondary school mathematics. Conventional instruction often relies on static resources, such as graph paper, rulers, and mirrors, which provide limited opportunities for learners to visualize and explore geometric transformations dynamically (Ayebale et al., 2020; Ubi et al., 2018; Zeinul & Silmi, 2022). Consequently, many learners experience difficulties in developing spatial reasoning and linking geometric representations with algebraic concepts (Kidega et al., 2023). Evidence from Uganda Certificate of Education examination scripts further shows persistent weaknesses in geometry, particularly in tasks requiring higher-order geometric reasoning (Ndungo et al., 2024).

CONTACT Issa Ndungo  ndungoissa@mmu.ac.ug  Mountains of the Moon University, Fort-Portal, Uganda

© 2026 The Author(s). Published by Informa UK Limited, trading as Taylor & Francis Group

This is an Open Access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. The terms on which this article has been published allow the posting of the Accepted Manuscript in a repository by the author(s) or with their consent.

The Van Hiele model, developed by Pierre and Dina Van Hiele, is recognized globally for its structured approach to geometric learning. It outlines five hierarchical levels of geometric thinking, each building upon the previous one, from basic visualization to more advanced formal deduction (Crowley, 1987; Vojkuvkova, 2012). Effective instruction that guides learners through these levels is crucial for their deep understanding of geometry concepts. Studies in diverse contexts have shown that traditional, teacher-centered instructional approaches often fail to facilitate this progression (Crowley, 1987; Machisi & Feza, 2021; Sert Celik & Kaleli Yilmaz, 2022; Tahani, 2016). Such approaches are prevalent in resource-limited settings, which are characteristic of many developing countries.

Consequently, many countries have shifted toward learner-centered pedagogies that emphasize active participation, hands-on learning, and problem-solving, aligning with the principles of constructivism. These pedagogies focus on knowledge building through experience and learner interaction with the environment (Allen, 2022). In countries such as Indonesia, South Africa, Nigeria, Pakistan, Sri Lanka, and Italy, integrating technology into geometry instruction has become a common practice, with positive outcomes reported in learners' engagement and understanding of abstract concepts (Ayebale et al., 2020; Ubi et al., 2018; Zeinul & Silmi, 2022). This allows learners to explore geometric relationships visually and interactively, fostering deeper conceptual understanding (Çavuş & Deniz, 2022; Mishra & Koehler, 2006; Sher et al., 2023; Wachira & Keengwe, 2011).

Recognizing these global trends, Uganda's education system has recently adopted a Competency-Based Curriculum (CBC) at the secondary level. This aims to shift from traditional, teacher-centered methods to learner-centered instruction, emphasizing problem-solving, critical thinking, and active participation by learners (NCDC, 2019a). However, this has been challenging for many teachers, who must adapt to long-established teaching practices. There is a need for instructional strategies that align with the CBC's goals and effectively support learners' progression through the Van Hiele levels (Kidega et al., 2023).

Van Hiele Phased Instruction (VHPI) offers a structured, research-based approach that has the potential to support the CBC's emphasis on learner-centered learning (Ndungo et al., 2024). It progressively guides learners through visualization, analysis, informal deduction, and more advanced reasoning stages (Vojkuvkova, 2012). By providing activities tailored to each stage of geometric reasoning, VHPI has shown promise in international contexts for improving learners' understanding of geometry. However, its effectiveness has not been thoroughly investigated in Uganda's educational context, where learners struggle with geometric reasoning, and teachers face challenges implementing the CBC (Kidega et al., 2023).

The Competency-Based Curriculum (CBC) advocates integrating technology to support learner-centred instruction, problem-solving, critical thinking, and active participation (NCDC, 2019a, 2019b). Within the Van Hiele instructional model, however, technology serves a pedagogical rather than merely technological role. GeoGebra enriches each instructional phase by supporting dynamic visualization during information, structured exploration in guided orientation, mathematical communication during Explicitation, independent investigation and immediate feedback in free orientation, and the integration of graphical, algebraic, and geometric representations during consolidation. Thus, Technology-Enhanced Van Hiele Phased Instruction (TVHPI) extends the conventional Van Hiele model by strengthening learning experiences within each instructional phase through purposeful use of GeoGebra rather than replacing its pedagogical principles. Previous studies have shown that technology-supported geometry instruction enhances learner engagement and conceptual understanding (Dahal et al., 2022; KÜÇÜK & GÜN, 2023; Nzaramyimana et al., 2021; Soheila & Rosemaliza, 2018; Uwurukundo et al., 2021). However, empirical evidence on how such integration supports learners' progression through the Van Hiele levels within Uganda's competency-based curriculum remains limited.

A related strand of our earlier work examined learners' experiences and attitudes under TVHPI and CVHPI within the context of Uganda's competency-based curriculum (Ndungo et al., 2025a, 2025b). The present paper builds on these findings by determining whether the two instructional approaches lead to measurable progression across the Van Hiele levels. By linking experience, and geometric reasoning, the study provides a concise yet comprehensive evaluation of the effectiveness of TVHPI and CVHPI in supporting conceptual development in transformation geometry.

Accordingly, the present study contributes to mathematics education in three important ways. First, it evaluates instructional effectiveness using learners' progression through the Van Hiele hierarchy, thereby providing a theoretically grounded measure of conceptual development beyond conventional achievement

outcomes. Second, it demonstrates how GeoGebra can be purposefully integrated within the instructional phases of the Van Hiele model, strengthening the theoretical connection between technology-enhanced learning and geometry instruction. Third, it provides empirical evidence from implementing Technology-Enhanced Van Hiele Phased Instruction within Uganda's Competency-Based Curriculum, extending current knowledge on learner-centred geometry instruction in resource-constrained educational settings.

Theoretical foundations

This study is grounded in the Van Hiele theory of geometric thinking, which explains how learners progressively develop geometric reasoning through hierarchical levels of thinking and corresponding phases of instruction (Crowley, 1987; Vojkuvkova, 2012). A central proposition of the theory is that progression across the Van Hiele levels depends not simply on learners' age or exposure to geometry, but on carefully sequenced instructional experiences that match their current level of reasoning. Consequently, learners experience difficulties when instruction requires reasoning beyond their cognitive readiness (Abdullah & Zakaria, 2013b; Moru et al., 2020).

The instructional principles of the Van Hiele model are consistent with cognitive and social constructivist perspectives on learning. Cognitive constructivism emphasises that learners actively construct knowledge through meaningful engagement with mathematical ideas, while social constructivism highlights the importance of guided interaction, discussion, and teacher scaffolding within the Zone of Proximal Development (Allen, 2022). These perspectives support the learner-centred philosophy of Uganda's Competency-Based Curriculum, where learners are expected to investigate, communicate, justify, and apply mathematical ideas rather than memorize procedures (NCDC, 2019a, 2019b).

In the present study, Conventional Van Hiele Phased Instruction (CVHPI) operationalizes these theoretical principles using traditional instructional resources, whereas Technology-Enhanced Van Hiele Phased Instruction (TVHPI) employs GeoGebra to strengthen learners' exploration, visualisation, reasoning, and conceptual development while preserving the pedagogical structure of the Van Hiele instructional phases. Thus, technology is viewed as a means of strengthening theory-driven instruction rather than as an instructional approach independent of the Van Hiele model.

Conventional Van Hiele's Phases Instruction (CVHPI)

The Conventional Van Hiele Phased Instruction is a structured framework for teaching geometry that supports learners' understanding and progression through the Van Hiele levels. This model guides educators in designing lessons that help students advance through increasingly complex levels of understanding with teacher support. It has five phases: (1) Inquiry, where students explore concepts; (2) Directed Orientation, focusing on specific properties; (3) Explication, where learners articulate understanding; (4) Free Orientation, applying knowledge to complex problems; and (5) Integration, consolidating concepts for deeper comprehension. Although the Van Hiele theory comprises five hierarchical levels, this study focused on learners' progression from Visualization to Deduction because the Senior Three transformation geometry curriculum does not extend to the Rigor level.

Conventional Van Hiele Phased Instruction provides the pedagogical structure through which learners progress from intuitive recognition of geometric forms to increasingly sophisticated levels of reasoning. Rather than presenting geometric concepts as isolated procedures, the instructional phases systematically guide learners through exploration, explanation, independent problem-solving, and conceptual integration. Previous studies have consistently reported that instruction organised according to these phases improves learners' geometric reasoning and conceptual understanding across different educational contexts (Abdullah & Zakaria, 2013b; MdYunus et al., 2019). Consequently, CVHPI provides the theoretical benchmark against which the additional contribution of technology-enhanced implementation can be evaluated.

Technology-Enhanced Van Hiele Phased Instructional (TVHPI)

Technology integration in education has expanded significantly, aligning with curriculum trends that emphasize active learning through interactive tools such as tablets, smartphones, and specialized

software. The use of ICT tools supports learners' integration into mathematics education, providing more significant learning opportunities and fostering engagement and discovery-based learning (Mosese & Ogbonnaya, 2021).

Various software tools, including GeoGebra, MATLAB, Mathematica, Maple, and Geometer's Sketchpad, have been developed specifically for teaching mathematics (Praveen & Leong, 2013). GeoGebra is one of the most widely studied tools, although its use remains limited in certain regions, such as Uganda (Mosese & Ogbonnaya, 2021). GeoGebra is a free, open-source dynamic geometry software that offers visualization, algebra, and spreadsheet functions. Its accessibility makes it suitable for schools, allowing multiple installations at no cost (Soheila & Rosemaliza, 2018). It allows students to simultaneously view numerical and geometric representations, facilitating a deeper understanding (Yildiz & Baltaci, 2016).

Technology-Enhanced Van Hiele Phased Instruction extends the conventional instructional model by embedding GeoGebra within each instructional phase of the Van Hiele framework. Dynamic visualisation enables learners to manipulate geometric objects, observe invariant properties, and test geometric relationships in real time. Immediate visual feedback supports learners in verifying conjectures, identifying misconceptions, and refining mathematical reasoning during exploration. By linking graphical and algebraic representations, GeoGebra also facilitates conceptual integration across successive Van Hiele levels. Previous studies have demonstrated the educational benefits of GeoGebra for improving visualisation, spatial reasoning, and learner engagement (Celen, 2020; Mollakuqe et al., 2020; Mudaly & Fletcher, 2019; Uwurukundo et al., 2021). However, the present study extends this literature by examining whether integrating GeoGebra within the pedagogical structure of Van Hiele Phased Instruction promotes greater progression through the Van Hiele hierarchy than implementing the same instructional model using conventional classroom resources. Figure 1 shows a GeoGebra environment.

GeoGebra tools enable users to dynamically adjust rotation angles and observe changes in real-time, thereby enhancing their understanding of rotational transformations through the visual representation of the rotation's center, angle, and direction. This interactive environment enables students to explore geometric transformations with precision and clarity. To better understand how these two instructional strategies differ in practice, the following section outlines key implementation differences between CVHPI and TVHPI in teaching transformation geometry.

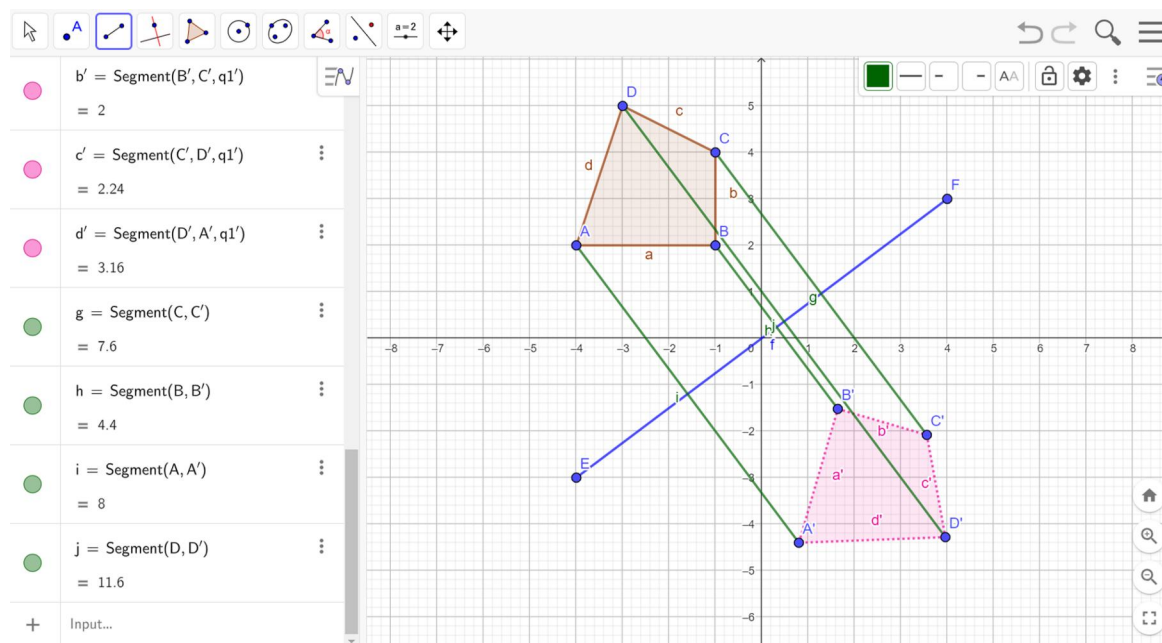


Figure 1. GeoGebra environment showing the rotation of a quadrilateral and the angle of rotation.

Implementing differences between CVHPI and TVHPI

The principal distinction between Conventional Van Hiele Phased Instruction (CVHPI) and Technology-Enhanced Van Hiele Phased Instruction (TVHPI) lies not in the instructional sequence, as both follow the five Van Hiele instructional phases, but in how learning experiences are delivered. In CVHPI, teachers rely on physical resources such as graph paper, rulers, tracing paper, mirrors, and blackboard demonstrations to guide learners through each phase. In contrast, TVHPI integrates GeoGebra to enrich learning while preserving the pedagogical intent of the Van Hiele model.

During the Information phase, GeoGebra introduces transformation concepts through dynamic visual demonstrations that activate learners' prior knowledge. In Guided Orientation, learners manipulate geometric figures, observe invariant properties of transformations, and complete structured exploration tasks using GeoGebra. During Explicitation, learners use dynamic representations to describe and justify geometric relationships using appropriate mathematical language. In Free Orientation, learners independently investigate multiple transformation scenarios, test conjectures, and receive immediate visual feedback from the software. Finally, during the Integration phase, learners consolidate their understanding by linking graphical, algebraic, and geometric representations to form coherent conceptual structures.

Compared with CVHPI, which relies on static representations, teacher-directed demonstrations, and manual feedback, TVHPI provides dynamic visualization, immediate feedback, interactive exploration, and greater learner autonomy while maintaining the same instructional phases. Consequently, the teacher's role shifts from primarily demonstrating procedures to facilitating inquiry, reasoning, and conceptual understanding. Despite these pedagogical advantages, empirical evidence on whether integrating GeoGebra within the Van Hiele instructional phases enhances learners' progression through the Van Hiele levels in Uganda's Competency-Based Curriculum remains limited, thereby motivating the present study.

Problem statement

In Uganda's lower secondary curriculum, transformation geometry is taught progressively from Senior 1 to Senior 3, aligning well with the Van Hiele levels of geometric reasoning (Ndungo et al., 2024). However, learners struggle with conceptual understanding and spatial reasoning, particularly as tasks become more abstract in later grades (Ndungo et al., 2024, 2025a; UNEB, 2016, 2021). Although Van Hiele Phased Instruction (VHPI) offers a structured pedagogical model, and technology like GeoGebra provides dynamic visualization opportunities, limited research has examined how these strategies influence learners' progression through the Van Hiele levels in actual classroom settings.

Furthermore, Uganda implements a competency-based curriculum emphasizing learner-centered, technology-enhanced instruction. Nevertheless, its implementation remains challenging as teachers transition from traditional, teacher-centered methods to constructivist, learner-centered strategies (Kidega et al., 2023). There is an urgent need to establish whether integrating tools like GeoGebra within CVHPI can support conceptual mastery in transformation geometry. This study seeks to address this gap by comparing the effects of CVHPI and TVHPI on students' advancement through the Van Hiele levels. To determine which approach more effectively advances learners through the Van Hiele levels in transformation geometry within the context of the CBC framework.

Purpose of the study and research hypotheses

The study's purpose is to compare the effect of instructional strategies on learners' progression through Van Hiele's levels, addressing two hypotheses: **H₀₁**: Learners exposed to TVHPI or CVHPI will not exhibit statistically significant progression in Van Hiele levels from pre-test to post-test, and **H₀₂**: There is no significant difference in Van Hiele levels progression between learners taught using TVHPI and those taught using CVHPI.

Scope and justification of the study

This study was conducted in six secondary schools in Midwestern Uganda, spanning six months. It explicitly addresses transformation geometry concepts, such as reflection, rotation, transformation matrices, and enlargement, to evaluate how each instructional strategy facilitates learners' advancement through these levels.

This study provides valuable insights into the effectiveness of TVHPI compared to CVHPI in advancing learners through the Van Hiele levels of geometric reasoning. It helps educators and policymakers understand the benefits of integrating technology into geometry instruction within Uganda's CBC framework. The findings can guide teacher training, support more effective classroom strategies, and improve student outcomes in transforming geometry and mathematics. Additionally, the study contributes to global discussions on technology-enhanced learning, offering evidence of its impact in resource-limited settings.

Methods

Study design

This study follows a *post-positivist* research paradigm, which acknowledges that while objectivity in research is the goal, complete objectivity is difficult to attain. Post-positivism emphasizes the use of rigorous empirical methods and statistical analysis to understand and measure reality, while acknowledging the influence of context and the limitations of measurement. This approach aligns with the study's goal of evaluating the effectiveness of two instructional methods, TVHPI and CVHPI, on learners' progression through the Van Hiele levels of geometric reasoning. The study employed a *quantitative* approach, utilizing a quasi-experimental design that incorporated pre-test and post-test measures to assess changes in learners' understanding of transformation geometry. This approach enables the examination of causality between the instructional methods and the observed changes in students' geometric reasoning, adhering to the principles of post-positivist inquiry (Park et al., 2020; Siddiqui, 2019).

Sample selection and participants

Schools in rural and urban districts in Midwestern Uganda were matched based on inclusion/exclusion criteria. The criteria were based on the availability of a functional computer laboratory, multiple streams in S.3 (the third-year class at secondary level, with learners aged 16 to 17), and the willingness of the school administration to participate. This process resulted in three sets of matched schools from the rural and two sets from the urban districts. One triple was randomly selected from each district, resulting in six schools (three rural and three urban) being included in the study. Within each school, two S.3 streams were randomly selected and assigned to either CVHPI (Experiment 1) or TVHPI (Experiment 2).

Intact classroom streams were used because reorganizing learners into new instructional groups was not feasible within normal school timetables and would have disrupted established teaching arrangements. Random assignment was therefore performed at the classroom-stream level rather than at the individual learner level, a common practice in school-based quasi-experimental research. To minimise selection bias, both intervention groups were drawn from the same school, taught the same curriculum content over the same period, and were instructed by the same classroom teacher using standardised lesson plans.

The two interventions were implemented in the same school, and group content sharing had no significant effect as the hands-on, technology-enhanced experience remained exclusive to the TVHPI group.

The study initially involved 651 S.3 learners, comprising 317 from rural schools and 334 from urban schools. However, due to dropouts and incomplete test data, the final analysis included 483 learners. The sample size was determined using G*Power software (version 3.1.9.7) with an effect size of 0.5, a power of 0.95, and a two-tailed test, resulting in a required minimum sample size of 222 per group, totaling 444 learners. The final sample exceeded the minimum required sample, ensuring robust statistical analysis (Faul et al., 2007; Novikov & Novikov, 2013).

Procedures

The procedures for this study closely followed the same structured approach used in our earlier investigations (Author 1, Author 2, & Author 3, 2025c; Author 1, Author 2, Author 3, et al., 2025). Before implementation, participating teachers attended a two-day training workshop designed to ensure consistent delivery of both instructional approaches. The training covered the theoretical foundations of the Van Hiele model, implementation of the five instructional phases, standardised lesson planning, learner facilitation strategies, and assessment procedures. Teachers responsible for TVHPI additionally received practical training in GeoGebra, including classroom management within the computer laboratory and the integration of dynamic visualisation into each instructional phase. To minimise teacher effects, the same teacher taught both instructional groups within each participating school.

To ensure consistency, the first author and participating teachers collaboratively developed standardised lesson plans aligned with all learning outcomes. Pretests were administered (in May 2024) to establish baseline Van Hiele levels before the six-week intervention (from 3, June 2024 to 12, July 2024), during which both groups received 24 lessons taught by the same teacher and monitored for fidelity. Fidelity of implementation was monitored throughout the intervention by the first author using structured classroom observations based on the planned lesson sequence. Particular attention was given to adherence to the instructional phases, lesson objectives, pacing, learner activities, and appropriate use of GeoGebra in the TVHPI group. Any minor deviations identified during observations were discussed with teachers to ensure consistency across subsequent lessons.

TVHPI lessons were delivered in the computer laboratory using GeoGebra, while CVHPI lessons were conducted in regular classrooms using traditional tools. After instruction, post-tests measured changes in Van Hiele levels, and, for ethical fairness, CVHPI learners were later given one week of access to GeoGebra.

Data collection tools and data analysis methods

Quantitative data were collected before (pre-test) and after the intervention (post-test) using the Van Hiele Geometry Achievement Test (VGAT), developed to assess learners' geometric reasoning based on the Van Hiele model. Two parallel forms of the instrument, VGAT1 (pre-test) and VGAT2 (post-test), each comprised 20 equivalent items covering key transformation geometry concepts. The items assessed learners from Visualization to Deduction, with five items representing each of the first four Van Hiele levels (Visualization, Analysis, Abstraction, and Deduction). Although the Van Hiele theory comprises five hierarchical levels, the Rigor level was not assessed because it lies beyond the transformation geometry content prescribed for Senior Three in Uganda's Competency-Based Curriculum.

Five mathematics education specialists and experienced secondary school teachers independently reviewed both versions of the VGAT to evaluate content relevance, curriculum alignment, clarity, and appropriateness for measuring progression across the Van Hiele hierarchy. Their recommendations informed revisions to item wording, sequencing, and level alignment before pilot testing. A pilot study involving 30 Senior Three learners was then conducted to evaluate item clarity, administration procedures, and score consistency. Construct validity was strengthened by organising items hierarchically according to the cognitive progression proposed by the Van Hiele model, ensuring that advancement to a higher level depended upon demonstrated mastery of preceding levels. Internal consistency reliability was acceptable (Cronbach's $\alpha = 0.781$), indicating satisfactory consistency for measuring learners' geometric reasoning. Together, expert validation, pilot testing, hierarchical item construction, and reliability analysis provided evidence that the VGAT was appropriate for assessing learners' progression through the operational Van Hiele levels used in this study.

Learners were scored on a scale of 0 to 5 points at each level. Progression through the levels was cumulative, requiring a minimum score of 3 out of 5 points for a given level and all preceding levels. This ensured that mastery of earlier levels was necessary for advancement, aligning with the hierarchical nature of the Van Hiele model. The scoring rubric developed by Van Hiele evaluated accuracy, cognitive alignment, and logical progression, providing a detailed assessment of learners' geometric reasoning

and progression through the levels (Vojkuvkova, 2012). Figure 2 shows the algorithm followed when placing learners at different Van Hiele levels following the scoring rubric.

Learners start at Level 0 (pre-visualization) and progress sequentially through Levels 1 to 4, advancing only if they correctly answer at least 3 out of 5 items ($\geq 60\%$) at each level. Failure at any level stops progression, and the last successfully achieved level is recorded as the learner's placement. This diagnostic process identifies learners' current geometric reasoning stage and guides teachers in providing targeted, level-appropriate instruction.

The difference in Van Hiele levels (Diff-VHL) was calculated as *the post-test value minus the pre-test value*, capturing the progression or regression between the pre-test and post-test. Values of Diff-VHL ranged from -4 to 4 , where 0 indicates no progress (the learner stayed at the same level), positive values (1 to 4) represent improvement (e.g. a value of 1 indicates an advance of one level, while 4 indicates an advance from Pre-Visualization to Deduction), and negative values (-1 to -4 indicate a decline (e.g. -1 means the learner regressed by one level, while -4 indicates a rare, significant drop from Deduction back to Pre-Visualization)).

Data were analysed using descriptive and inferential statistics. Descriptive statistics summarised learners' distribution across the operational Van Hiele levels before and after the intervention. Since the dependent variable (Van Hiele level) was ordinal and the instructional group was binary ($CVHPI = 0$, $TVHPI = 1$), non-parametric and ordinal regression techniques were employed. The Wilcoxon Signed-Rank Test was used to assess within-group changes in learners' Van Hiele levels from pre-test to post-test. An ordinal regression model was fitted to examine the effect of instructional strategy on learners' likelihood of attaining higher Van Hiele levels at post-test. Finally, Difference-in-Differences (DiD) analysis was conducted to compare changes in mean Van Hiele levels between the two instructional groups over time.

Because the intervention was implemented using intact classroom streams, clustering was assessed using an unconditional random-intercept model. The estimated interclass correlation coefficient (ICC)

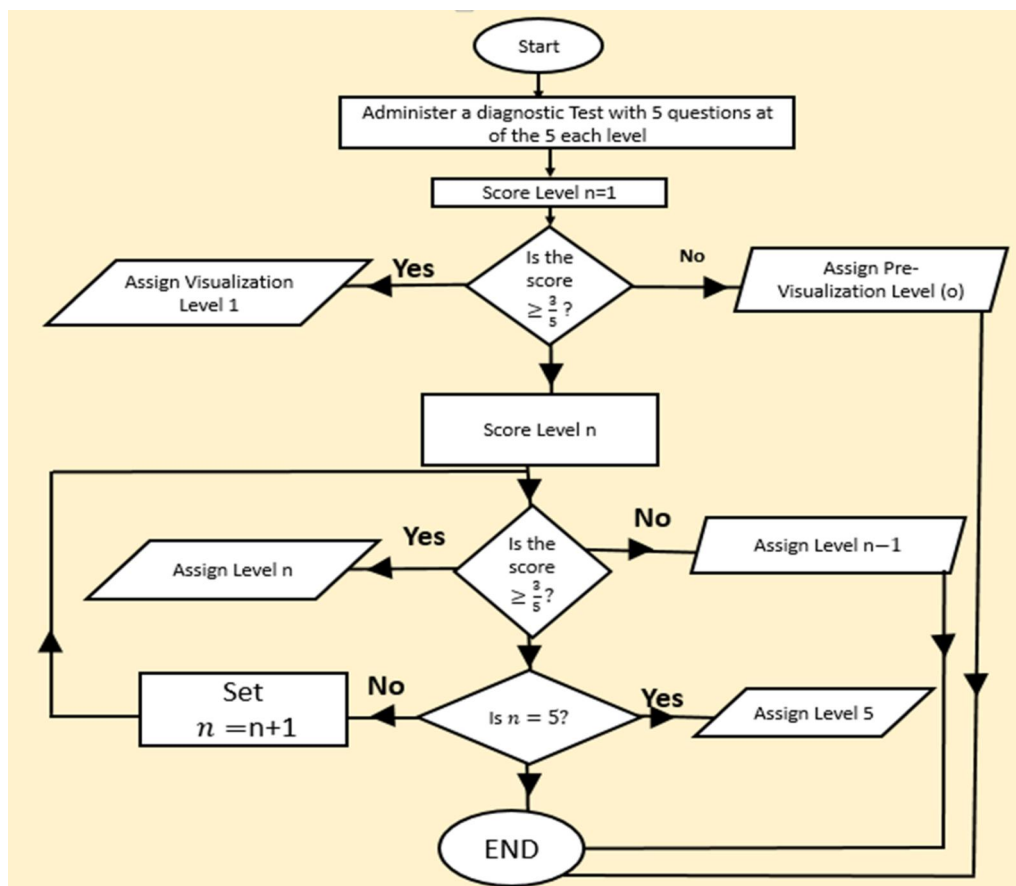


Figure 2. Algorithm for placing learners at different Van Hiele levels.

was 0.043, indicating that 4.3% of the variation in post-test Van Hiele levels was attributable to differences between classroom streams. Given the modest level of clustering, the primary analyses controlled for baseline Van Hiele levels while comparing the instructional approaches.

Ethical considerations

The study used established ethical guidelines to safeguard participants' rights, ensure data integrity, and promote objective reporting. Ethical clearance was obtained from the research ethics committee of Mbarara University of Science and Technology (MUST-2024-1519, Date: 03/07/2024) and the Uganda National Council of Science and Technology (SS2857ES, Date: 06/08/2024). Additional written permissions were secured from local education offices and participating schools. Written informed consent was obtained from all adult participants, while assent and consent from parents or guardians were obtained for minors. Participant confidentiality and anonymity were preserved by coding all data and restricting access to authorized personnel only.

To enhance accuracy and transparency, the lead researcher performed the data analysis, and all analytic procedures, including scripts and raw outputs, were independently reviewed by additional researchers. Classroom teachers who supported intervention delivery also assisted in verifying the interpretation of data. Furthermore, to promote equitable access to digital resources, students in the non-technology group were provided a one-week opportunity to explore GeoGebra after the intervention, ensuring fairness in technological exposure.

Results

This study aimed to investigate how learners' geometric reasoning, as measured by Van Hiele levels, developed under two instructional strategies: Conventional Van Hiele Phased Instruction and Technology-Enhanced Van Hiele Phased Instruction. The results presented in this section address two key hypotheses: whether each strategy significantly influenced learner progression, and whether there were meaningful differences between the two instructional approaches in promoting conceptual understanding in transformation geometry; thereby identifying which of the two strategies is superior in supporting learners' progression.

Following the preliminary checks for reliability, descriptive statistics were used to map learners' distribution across the Van Hiele levels before and after instruction. Table 1 illustrates the distribution of learners across the different Van Hiele levels before and after instruction in both instructional groups.

Table 1 presents the number of learners and percentage distribution in each Van Hiele level by instructional group in the pre-test and post-test. The Van Hiele levels include pre-visualization, representing students who are below the first formal Van Hiele level at the pre-test. The visualization, analysis, abstraction, and deduction levels are the first four Van Hiele levels; based on the study context, the last (fifth) Van Hiele level was out of scope for this study.

In CVHPI, 44.5% began at pre-visualization, decreasing to 9.0% after the intervention, with notable gains in visualization (30.6–41.6%) and moderate increases in analysis, abstraction, and deduction. Similarly, TVHPI showed a decline in pre-visualization (41.6–5.0%), with many learners advancing to analysis (38.7%) and some reaching abstraction and deduction. The unexpected drop in visualization

Table 1. Frequency distribution for pre-VHL and post-VHL scores by group.

Van hiele levels	CVHPI		TVHPI	
	Pre-VHL	Post_VHL	Pre-VHL	Post_VHL
0. Pre visualization	109 (44.5%)	22 (9.0%)	99 (41.6%)	12 (5.0%)
1. Visualization	75 (30.6%)	102 (41.6%)	80 (33.6%)	70 (29.4%)
2. Analysis	43 (17.6%)	82 (33.5%)	40 (16.8%)	92 (38.7%)
3. Abstraction	18 (7.4%)	37 (15.1)	19 (8.0%)	61 (25.6%)
4. Deduction	–	2 (0.8%)	–	3 (1.3%)

Table comparing the frequency and percentage distribution of learners across Van Hiele levels before and after CVHPI and TVHPI. Both groups show movement from lower to higher Van Hiele levels after instruction, with the TVHPI group showing a larger proportion of learners reaching Analysis and Abstraction at post-test. Few learners reached Deduction in either group.

suggests movement directly into higher levels. Overall, both methods promoted progression, with most learners advancing to visualization and analysis, and fewer reaching abstraction or deduction.

To test the first hypothesis (H_{01}), the Wilcoxon Signed-Rank Test was employed. This test compared learners' Van Hiele levels before and after instruction within each group (from pre-test to post-test) to determine whether the observed progression within each instructional group was statistically significant. The results are summarized in Table 2.

The results in Table 2 indicate that both instructional groups made statistically significant gains in Van Hiele levels from pre-test to post-test. For CVHPI, the Wilcoxon statistic of 1164.500 ($p < 0.001$) shows meaningful progression, leading to rejection of the first part of H_{01} . Similarly, TVHPI demonstrated significant gains with a statistic of 133.000 ($p < 0.001$), rejecting the second part of H_{01} and confirming the positive effect of technology. Thus, both instructional approaches contributed to the improved geometric reasoning.

To determine whether learners' progression through the Van Hiele hierarchy differed between instructional groups (H_{02}), an ordinal regression model was fitted using post-test Van Hiele level as the dependent variable and instructional group as the predictor. An ordinal regression approach was appropriate because the outcome variable comprised ordered Van Hiele levels, allowing estimation of the likelihood of learners attaining higher levels of geometric reasoning following the intervention. The results are presented in Table 3.

Table 3 indicates that instructional strategy significantly predicted learners' post-test Van Hiele levels. Learners in the TVHPI group had a positive and statistically significant coefficient ($\beta = 0.661$, $SE = 0.169$, $z = 3.91$, $p < 0.001$). The corresponding odds ratio ($OR = 1.94$) indicates that learners receiving Technology-Enhanced Van Hiele Phased Instruction were approximately 1.94 times more likely to attain higher Van Hiele levels than learners receiving Conventional Van Hiele Phased Instruction. The threshold parameters define the boundaries between adjacent Van Hiele levels within the ordinal regression model and facilitate estimation of learners' progression across the ordered categories. Overall, these findings indicate that instructional approach significantly influenced learners' progression through the Van Hiele hierarchy, leading to the rejection of H_{02} . To estimate the magnitude of improvement over time between the two instructional groups, a Difference-in-Differences analysis was subsequently conducted (Table 4), with the trends illustrated in Figure 3.

Table 4 shows the OLR model results incorporating group, time, and their interaction as ordered predictors. At pre-test, the TVHPI group had slightly higher odds of being at a higher Van Hiele level ($OR = 1.09$), but the effect was not significant ($\beta = 0.085$, $p = 0.623$), indicating both groups started equally. At post-test, learners, regardless of group, had significantly higher odds of progressing ($OR = 4.12$, $\beta = 1.416$, $p < 0.001$). However, TVHPI learners demonstrated a substantially greater likelihood of advancing, with a 65% higher chance than CVHPI learners, supporting rejection of H_{02} .

Table 2. Wilcoxon Signed-Rank test results for within-group progression.

Instructional group	Test statistic (W)	p-Value	Interpretation
CVHPI	1164.5000	<0.001	Significant change (reject H_{01})
TVHPI	133.0000	<0.001	Significant change (reject H_{01})

Table presenting Wilcoxon Signed-Rank Test results for pre-test to post-test changes in Van Hiele levels within the CVHPI and TVHPI groups. Both instructional groups showed statistically significant progression, with p -values below .001.

Table 3. Ordered regression model – effect of instructional group on post-test Van Hiele levels.

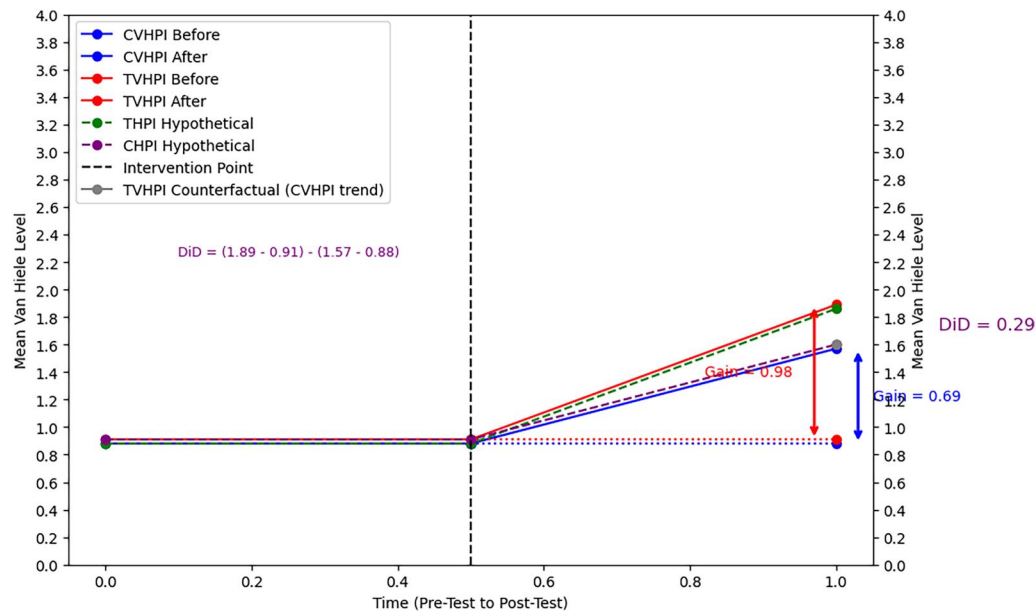
Coefficient	Estimate (β)	Std. Error	z-Value	p-Value	Odds ratio	Interpretation
Instructional Group (TVHPI)	0.661	0.169	3.91	0.000	1.94	Learners in the TVHPI group are 1.94 times more likely to reach a higher Van Hiele level.
Threshold 0/1	-2.302	0.190	-12.10	0.000	0.10	Cut-point between Level 0 and Level 1
Threshold $\frac{1}{2}$	0.843	0.075	11.24	0.000	2.32	Cut-point between Level 1 and Level 2
Threshold $\frac{2}{3}$	0.496	0.068	7.34	0.000	1.64	Cut-point between Level 2 and Level 3
Threshold $\frac{3}{4}$	1.187	0.135	8.80	0.000	3.28	Cut-point between Level 3 and Level 4

Table presenting the ordered regression results for the association between instructional group and post-test Van Hiele levels. Learners in the TVHPI group had higher odds of attaining a higher Van Hiele level than learners in the CVHPI group (odds ratio = 1.94, $p < .001$). The table also presents threshold estimates separating successive Van Hiele levels.

Table 4. DiD Model results – ordinal logistic regression of Van Hiele Levels.

Predictor	Coefficient (β)	Std. Error	z-Value	p-Value	Odds ratio	Interpretation
Group (TVHPI)	0.085	0.173	0.492	0.623	1.09	No significant difference in the pre-test
Time (Post)	1.416	0.172	8.255	0.000	4.12	Significant progression over time
Interaction	0.503	0.238	2.119	0.034	1.65	TVHPI led to a $1.65 \times$ greater gain in Van Hiele levels

Table presenting Difference-in-Differences ordinal logistic regression results comparing changes in Van Hiele levels between the CVHPI and TVHPI groups over time. The groups did not differ significantly at pre-test, both showed significant progression over time, and the significant group-by-time interaction indicates greater progression in the TVHPI group relative to the CVHPI group.

**Figure 3.** The Difference-in-Differences (DiD) graph.

On the other hand, [Figure 3](#) illustrates the DiD trends for Van Hiele levels under TVHPI and CVHPI. Both groups show upward progression, with TVHPI increasing more sharply. The nearly identical pre-test means minimize baseline bias, while hypothetical swapped-baseline lines confirm that TVHPI would still outperform CVHPI, indicating limited influence of initial levels on effectiveness. The DiD gap, shown by the difference between the red and blue arrows, reveals greater gains for TVHPI, indicating stronger conceptual development in transformation geometry. These graphical results, consistent with statistical findings, support rejecting H_{02} and confirm TVHPI's superior effectiveness in advancing learners' geometric reasoning.

Discussion

The findings indicate that both instructional methods, Technology-Enhanced Van Hiele Phased Instruction and Conventional Van Hiele Phased Instruction, effectively facilitated learners' progression through Van Hiele levels in transformation geometry, confirming that both methods significantly improve geometric reasoning from pre-test to post-test. This finding aligns with the foundational principles of the Van Hiele model, which posits that structured, phased instruction supports incremental cognitive development in geometry (Pujawan et al., 2020; Sert Celik & Kaleli Yilmaz, 2022; Vojkuvkova, 2012). The results further suggest that the two approaches could be leveraged in different contexts; for example, in areas where technology is limited or inaccessible, the conventional Van Hiele phased instruction can be used as a structured, learner-centered strategy. Since both methods significantly improve learners' geometric reasoning, educators can confidently adopt Van Hiele's phased instruction as a core approach to geometry education, regardless of the specific tools used as recommended in Ndungo, Balimuttajjo, Akugizibwe, et al. (2025).

The success of both TVHPI and CVHPI in advancing learners' understanding reflects the inherent strengths of the Van Hiele phased instruction model, which organizes learning into structured phases

starting from Visualization and progressing to Rigor. The TVHPI is based on the VHPI, which has been found to support learner progression, as reflected in the current study's findings, providing supporting evidence that VHPI is a practical approach (Abdullah & Zakaria, 2013a; Demir et al., 2023; Naufal et al., 2021; Senk et al., 2022; Tahani, 2016).

This finding highlights the versatility and robustness of the Van Hiele model, regardless of the specific instructional tools employed. It resonates with cognitive-developmental theories, such as Bruner's constructivist approach, which emphasizes the importance of a structured learning sequence that encourages students to construct knowledge at each stage, actively emphasized by Xu et al. (2023), providing structured guidance through visualization, analysis, and, ultimately, abstraction (Scott & Palincsar, 2021).

Although both strategies facilitated progression, the analysis further demonstrated that Technology-Enhanced Van Hiele Phased Instruction is significantly more effective than Conventional Van Hiele Phased Instruction in advancing learners through Van Hiele levels of geometric understanding. This is possibly due to the dynamic nature of the TVHPI, where learners use GeoGebra to manipulate transformation shapes, enhancing their conceptual understanding. This is consistent with research suggesting that technology supports more dynamic and interactive learning environments, which are particularly effective for advancing spatial and abstract reasoning (Clements & Sarama, 2021). It also aligns with similar studies that investigated the use of GeoGebra in teaching different areas of mathematics, most of which found results in favor of GeoGebra in supporting learners' progression in VHL (Bekene, 2020; Dahal et al., 2022; Graeme Welch & Ponce Campuzano, 2025; Mudaly & Fletcher, 2019; Praveen & Leong, 2013; Soheila & Rosemaliza, 2018; Tamam & Dasari, 2021; Uwurukundo et al., 2022).

This finding highlights the added value of integrating technology into the Van Hiele phased instructional approach, thereby improving the overall effectiveness of the model. This is in support of existing studies, such as Dahal et al. (2019), KÜÇÜK and GÜN (2023), Soheila and Rosemaliza (2018), Zengin (2017), and Zulnaidi and Zakaria (2012), that found similar findings about integrating technology into the Van Hiele phased instruction. Technology provides learners with immediate feedback at each level, from Visualization to Deduction, allowing them to observe the outcomes of transformations directly (Ndungo, Nazziwa, et al., 2025). This aligns with constructivist principles, which posit that learners build knowledge through active exploration (Allen, 2022).

While this study's findings largely reinforce existing literature, some differences emerge in the effectiveness of technology-enhanced learning for all students. For instance, although Battista (2001) supports the role of technology in advancing spatial reasoning, other studies, such as Hoyles and Noss (2009), caution that not all learners benefit equally from technology, particularly in under-resourced settings. These findings highlight the need for further research on contextual factors that influence technology integration, an area crucial for CBC implementation in diverse Ugandan classrooms and beyond. The findings indicate that TVHPI aligns with CBC's learner-centered approach, promoting active, exploratory learning that constructivist theory champions as essential for deep conceptual understanding.

In General, this study addresses critical gaps in Uganda's secondary geometry education, particularly learners' challenges with progressing through the Van Hiele levels (VHL) of geometric understanding. By leveraging interactive technology, TVHPI enhances cognitive scaffolding through real-time visualization and feedback, providing a more engaging learning experience that promotes deeper understanding (Sinclair et al., 2012). Nonetheless, professional development in integrating technology with phased instruction is essential for optimal implementation, particularly in under-resourced areas, to maximize the benefits of TVHPI and ensure sustainable improvements in diverse Ugandan educational settings (Hegedus et al., 2015). The effectiveness of phased, structured instruction, evident in this study, aligns with the Van Hiele model's cognitive framework and supports its integration within Uganda's Competency-Based Curriculum (CBC), which emphasizes learner-centered and problem-solving strategies.

An alternative explanation for the superior performance of the TVHPI group is the possibility of a novelty effect associated with the introduction of GeoGebra. Learners may initially demonstrate greater motivation or engagement simply because the technology represents a new learning experience. Several measures were taken to minimise this possibility, including introductory sessions that familiarized learners with the software before formal instruction, implementation over a six-week instructional period rather than a single lesson, and maintaining identical lesson objectives and instructional phases across both groups. Nevertheless, the possibility that novelty contributed to some of the observed gains cannot

be completely excluded and should be considered when interpreting the findings. Future longitudinal studies could investigate whether these learning gains are sustained once the novelty of technology has diminished.

Limitations of the study

The study has some limitations that could inform future research. It focused on a specific region and time frame, which may limit the generalizability of the findings to other contexts. Although it effectively compared instructional methods, variations in how teachers applied them across schools could introduce differences in outcomes. Furthermore, the study did not assess the long-term retention of learning gains, leaving room for future studies to explore the sustained impacts.

Conclusions and recommendations

The findings indicate that Technology-Enhanced Van Hiele Phased Instruction was associated with greater progression through the Van Hiele levels than Conventional Van Hiele Phased Instruction within the context of this study. Although the results support the integration of technology in competency-based geometry instruction, they should be interpreted in light of the quasi-experimental design and the contextual characteristics of the participating schools. Further studies across different educational settings are recommended to confirm and extend these findings.

It is therefore recommended that educators adopt TVHPI, supported by appropriate training, and that curriculum developers embed such strategies within CBC implementation frameworks to promote structured, conceptual learning in geometry.

Acknowledgments

The authors gratefully acknowledge the participating schools, mathematics teachers, and learners for their cooperation and commitment throughout the study. We also appreciate the school administrators for facilitating the implementation of the intervention and the mathematics education experts who contributed to the validation of the research instruments.

Author contributions

CRedit: **Issa Ndungu**: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Resources, Software, Writing – original draft, Writing – review & editing; **Sudi Balimuttajjo**: Conceptualization, Validation; **Edwin Akugizibwe**: Methodology, Writing – original draft; **Charles Magoba Muwonge**: Conceptualization, Validation.

Ethical approval

The study was conducted in accordance with the ethical principles of the Declaration of Helsinki.

Informed consent

Written informed consent was obtained from all participants aged 18 years and above. Minors' parents or legal guardians provided consent, and assent was obtained from the students. All participants voluntarily agreed to partake in the research after receiving a comprehensive explanation of the study's aims, procedures, potential risks, and benefits.

Disclosure statement

No potential conflict of interest was reported by the author(s).

ORCID

Issa Ndungo  <http://orcid.org/0000-0003-0492-764X>

Data availability statement

The data that support the findings of this study are openly available and can be accessed through this link: <http://bit.ly/4lw31Fv>

References

- Abdullah, A., & Zakaria, E. (2013a). Enhancing students' level of geometric thinking through Van Hiele's Phase-based Learning. *Indian Journal of Science and Technology*, 6(5), 1–15. <https://doi.org/10.17485/ijst/2013/v6i5.13>
- Abdullah, A., & Zakaria, E. (2013b). The effects of Van Hiele's phases of learning geometry on students' degree of acquisition of Van Hiele Levels. *Procedia - Social and Behavioral Sciences*, 102(Ifee 2012), 251–266. <https://doi.org/10.1016/j.sbspro.2013.10.740>
- Allen, A. (2022). An introduction to constructivism: its theoretical roots and impact on contemporary education. *Journal of Learning Design and Leadership*, 1(1), 1–11.
- Asli, A., & Zsoldos-Marchis, I. (2021). Teaching applications of Mathematics in other disciplines: Teachers' opinion and practice. *Acta Didactica Napocensia*, 14(1), 142–150. <https://doi.org/10.24193/adn.14.1.11>
- Ayebale, L., Habaasa, G., & Tweheyo, S. (2020). Factors affecting students' achievement in mathematics in secondary schools in developing countries: A rapid systematic review. *Statistical Journal of the IAOS*, 36(S1), S73–S76. <https://doi.org/10.3233/sji-200713>
- Battista, M. T. (2001). A research-based perspective on teaching school geometry. In J. Brophy (Ed.), *Subject-specific instructional methods and activities* (Vol. 8, pp. 145–185). Emerald Group Publishing Limited. [https://doi.org/10.1016/S1479-3687\(01\)80026-2](https://doi.org/10.1016/S1479-3687(01)80026-2)
- Bekene, T. (2020). Implementation of GeoGebra a dynamic mathematical software for teaching and learning of calculus in Ethiopia Author: Tola Bekene, Wachemo University. *International Journal of Scientific & Engineering Research*, 11(9), 838–860. <https://doi.org/10.14299/ijser.2020.09.01>
- Çavuş, H., & Deniz, S. (2022). The effect of technology assisted teaching on success in mathematics and geometry: A meta-analysis study. *Participatory Educational Research*, 9(2), 358–397. <https://doi.org/10.17275/per.22.45.9.2>
- Celen, Y. (2020). Student opinions on the use of GeoGebra software in mathematics teaching. *Emerging Technologies in Computing*, 19(4), 84–88.
- Clements, D. H., & Sarama, J. (2021). Learning and teaching early math: The learning trajectories approach. In *Studies in mathematics thinking and learning* (2nd ed.). Routledge. <https://doi.org/10.56105/cjsae.v32i2.5596>
- Crowley, M. (1987). The van Hiele model of the development of geometric thought. *Learning and Teaching Geometry*, K-12, 1–16.
- Dahal, N., Pant, B. P., & Shrestha, D. (2019). Integration of GeoGebra in teaching and learning geometric transformation. *Journal of Mathematics and Statistical Science*, 5(December 2019), 323–332.
- Dahal, N., Pant, B. P., Shrestha, I. M., & Manandhar, N. K. (2022). Use of GeoGebra in Teaching and Learning Geometric Transformation in School Mathematics. *International Journal of Interactive Mobile Technologies (IJIM)*, 16(08), 65–78. <https://doi.org/10.3991/ijim.v16i08.29575>
- Demir, E., İlhan, O. A., & Sevgi, S. (2023). Investigation of seventh grade students van Hiele Geometric thinking levels in circle subject. *Bulletin of Education & Research*, 45(1), 95–118.
- Faul, F., Erdfelder, E., Lang, A.-G., & Buchner, A. (2007). G*Power 3: A flexible statistical power analysis program for the social, behavioral, and biomedical sciences. *Behavior Research Methods*, 39(2), 175–191. <https://doi.org/10.3758/bf03193146>
- Graeme Welch, B., & Ponce Campuzano, J. C. (2025). Applying matrix diagonalisation in the classroom with GeoGebra: Parametrising the intersection of a sphere and plane. *International Journal of Mathematical Education in Science and Technology*, 56(2), 332–348. <https://doi.org/10.1080/0020739X.2023.2233513>
- Hegedus, S. J., Dalton, S., & Tapper, J. R. (2015). The impact of technology-enhanced curriculum on learning advanced algebra in US high school classrooms. *Educational Technology Research and Development*, 63(2), 203–228. <https://doi.org/10.1007/s11423-015-9371-z>
- Hoyle, C., & Noss, R. (2009). The technological mediation of mathematics and its learning. *Human Development*, 52(2), 129–147. <https://doi.org/10.1159/000202730>
- Kidega, C., Song, Z., & Khaing, T. (2023). Factors affecting the implementation of competency-based curriculum in secondary schools in Uganda: A systematic literature review. *North American Academic Research*, 6(9), 84–102. <https://doi.org/10.5281/zenodo.10058421>
- Küçük, K., & Gün, Ö. (2023). The effects of GeoGebra-assisted transformation geometry instruction on student achievement, attitudes, and beliefs. *Journal of Computer and Educational Research*, 11(22), 672–690. <https://doi.org/10.18009/jcer.1324668>

- Machisi, E., & Feza, N. N. (2021). Van Hiele Theory-based instruction and grade 11 students' geometric proof competencies. *Contemporary Mathematics and Science Education*, 2(1), ep21007. <https://doi.org/10.30935/conmaths/9682>
- MdYunus, A. S., Ayub, A. F. M., & Hock, T. T. (2019). Geometric thinking of Malaysian elementary school students. *International Journal of Instruction*, 12(1), 1095–1112. <https://doi.org/10.29333/iji.2019.12170a>
- Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record: The Voice of Scholarship in Education*, 108(6), 1017–1054. <https://doi.org/10.1177/016146810610800610>
- Mollakuqe, V., Rexhepi, S., & Iseni, E. (2020). Incorporating Geogebra into teaching circle properties at high school level and it's comparison with the classical method of teaching. *International Electronic Journal of Mathematics Education*, 16(1), em0616. <https://doi.org/10.29333/iejme/9283>
- Moru, E. K., Malebanye, M., Morobe, N., & George, M. J. (2020). A Van Hiele Theory analysis for teaching volume of three-dimensional geometric shapes. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, 6(1), 17–31. <https://doi.org/10.23917/jramathedu.v6i1.11744>
- Mosese, N., & Ogbonnaya, U. I. (2021). GeoGebra and students' learning achievement in trigonometric functions graphs representations and interpretations. *Cypriot Journal of Educational Sciences*, 16(2), 827–846. <https://doi.org/10.18844/cjes.v16i2.5685>
- Mudaly, V., & Fletcher, T. (2019). The effectiveness of geogebra when teaching linear functions using the iPad. *Problems of Education in the 21st Century*, 77(1), 55–81. <https://doi.org/10.33225/pec/19.77.55>
- Naufal, M. A., Abdullah, A. H., Osman, S., Abu, M. S., Ihsan, H., & Rondiyah, R. (2021). Reviewing the Van Hiele model and the application of metacognition on geometric thinking. *International Journal of Evaluation and Research in Education (IJERE)*, 10(2), 597–605. <https://doi.org/10.11591/ijere.v10i2.21185>
- National Curriculum Development Centre (NCDC). (2019a). *Lower secondary curriculum, mathematics syllabus*. National Curriculum Development Centre, Uganda.
- National Curriculum Development Centre (NCDC). (2019b). *New lower secondary school curriculum framework*. National Curriculum Development Centre.
- Ndungo, I. (2024). An assessment of the alignment of transformation geometry with Van Hiele Levels in Uganda's lower secondary mathematics curriculum. *Turkish Journal of Mathematics Education*, 5(2), 33–41.
- Ndungo, I., Akugizibwe, E., & Balimuttajjo, S. (2024). Analyzing trends and suggested instructional strategies for Geometry education: Insights from Uganda Certificate of Education-Mathematics Examinations, 2016-2022. *African Journal of Teacher Education*, 13(2), 153–186. <https://doi.org/10.21083/ajote.v13i2.8106>
- Ndungo, I., Balimuttajjo, S., & Akugizibwe, E. (2025a). Conceptual and procedural trajectories in transformation geometry: A comparative study of technology-enhanced and conventional Van Hiele Phased Instruction. *International Journal of Education Technology and Science*, 5(2), 160–184. <https://doi.org/10.5281/zenodo.15499128>
- Ndungo, I., Balimuttajjo, S., & Akugizibwe, E. (2025b). Towards improved geometry instruction: Learners' experiences with technology-enhanced and conventional Van Hiele Phased Instruction. *The Electronic Journal for Research in Science & Mathematics Education*, 29(2), 33–56.
- Ndungo, I., Balimuttajjo, S., Akugizibwe, E., & Muwonge, C. M. (2025). Modeling learners' attitudinal shifts in transformation geometry under technology-enhanced and conventional Van Hiele Phased Instruction. *African Journal of Research in Mathematics, Science and Technology Education*, 29(2), 279–296. <https://doi.org/10.1080/18117295.2025.2529124>
- Ndungo, I., Nazziwa, C., Kabiswa, W., Opio, P., Akugizibwe, E., & Mpungu, K. (2025). Enhancing STEM instruction through indigenous materials and ICT integration: A critical assessment of teachers' knowledge, experiences, attitudes, and readiness. *East African Journal of Education Studies*, 8(2), 132–149. <https://doi.org/10.37284/eajes.8.2.2872>
- Novikov, A. M., & Novikov, D. A. (2013). *Research methodology: From philosophy of science to research design*. CRC Press. <https://doi.org/10.1201/b14562>
- Nzaramyimana, E., Mukandayambaje, E., Iyamuremye, L., Hakizumuremyi, V., & Ukobizaba, F. (2021). Effectiveness of GeoGebra towards students' active learning, performance and interest to learn mathematics'. *International Journal of Mathematics and Computer Research*, 09(10), 2423–2430. <https://doi.org/10.47191/ijmcr/v9i10.05>
- Park, Y. S., Konge, L., & Artino, A. R. (2020). The positivism paradigm of research. *Academic Medicine: Journal of the Association of American Medical Colleges*, 95(5), 690–694. <https://doi.org/10.1097/ACM.0000000000003093>
- Praveen, S., & Leong, K. E. (2013). Effectiveness of using Geogebra on students' understanding in learning circles. *The Malaysian Online Journal of Educational Technology*, 1(4), 1–11.
- Pujawan, I. G. N., Suryawan, I. P. P., & Prabawati, D. A. A. (2020). The effect of van hiele learning model on students' spatial abilities. *International Journal of Instruction*, 13(3), 461–474. <https://doi.org/10.29333/iji.2020.13332a>
- Scott, S., & Palincsar, A. (2021). Sociocultural theory. *Education.Com*, 1–10. <https://doi.org/10.1177/074193250002100201>
- Senk, S. L., Thompson, D. R., Chen, Y. H., Voogt, K., & Usiskin, Z. (2022). The Van Hiele Geometry Test: History, use, and suggestions for revisions. *University of Chicago School Mathematics Project*, 1–39. https://www.researchgate.net/publication/363118024%0Ahttps://www.researchgate.net/profile/Yi-Hsin-Chen-3/publication/363118024_The_Van_Hiele_Geometry_Test_History_Use_and_Suggestions_for_Revisions/links/630ea9721ddd4470212138cc/The-Van-Hiele-Geometry-Test

- Sert Celik, H., & Kaleli Yilmaz, G. (2022). Analysis of Van Hiele geometric thinking levels studies in Turkey: A meta-synthesis study conditions of the Creative Commons Attribution license (CC BY-NC-ND). *International Journal of Curriculum and Instruction*, 14(1), 473–501.
- Sher, M., Yasmeen, R., & Munawar, Z. (2023). The impact of technology integration on student engagement and achievement in mathematics education: A systematic review. *International Journal on Integrated Education*, 6(May), 222–232.
- Siddiqui, S. (2019). Research paradigms: Their assumptions and relevance. *International Journal of Research in Social Sciences*, 9(5), 254–265.
- Sinclair, N., Pimm, D., & Skelin, M. (2012). Developing essential understanding of geometry for teaching mathematics in grades 6–8. National Council of Teachers of Mathematics. <https://books.google.co.ug/books?id=ey8DtwAACAAJ>
- Soheila, B., & Rosemaliza, K. (2018). The intention to Use GeoGebra in the teaching of mathematics among Malaysian Teachers. *Malaysian Online Journal of Educational Technology*, 6(1), 109–115.
- Tahani, A. (2016). Effect of the Van Hiele Model in geometric concepts acquisition: The attitudes towards geometry and learning transfer effect of the first three grades students in Jordan. *International Education Studies*, 9(4), 87–98. <https://doi.org/10.5539/ies.v9n4p87>
- Tamam, B., & Dasari, D. (2021). The use of Geogebra software in teaching mathematics. *Journal of Physics: Conference Series*, 1882(1), 012042. <https://doi.org/10.1088/1742-6596/1882/1/012042>
- Ubi, E. E., Odiong, A. U., & Igiri, O. I. (2018). Geometry viewed as a difficult mathematics. *International Journal of Innovative Science and Research Technology*, 3(11), 251–255.
- UNEB. (2016). Uganda National Examinations Board, Report on work of the UCE candidates 2015. UNEB.
- UNEB. (2021). Uganda National Examinations Board, Report on the work of UCE candidates 2020. UNEB.
- Uwurukundo, M. S., Maniraho, J. F., & Tusiime, M. (2022). Enhancing students' attitudes in learning 3-dimension geometry using GeoGebra. *International Journal of Learning, Teaching and Educational Research*, 21(6), 286–303. <https://doi.org/10.26803/ijlter.21.6.17>
- Uwurukundo, M. S., Maniraho, J. F., & Tusiime, M. R. (2021). The effects of GeoGebra on students' attitudes towards learning geometry: A review of literature. *African Journal of Studies in Mathematics and Sciences*, 17(2), 127–138.
- Vojkuvkova, I. (2012). The van Hiele Model of geometric thinking. WDS'12 Proceedings of Contributed Papers, 1, 72–75. https://physics.mff.cuni.cz/wds/proc/pdf12/WDS12_112_m8_Vojkuvkova.pdf
- Wachira, P., & Keengwe, J. (2011). Technology integration barriers: Urban school mathematics teachers perspectives. *Journal of Science Education and Technology*, 20(1), 17–25. <https://doi.org/10.1007/s10956-010-9230-y>
- Xu, E., Wang, W., & Wang, Q. (2023). The effectiveness of collaborative problem solving in promoting students' critical thinking: A meta-analysis based on empirical literature. *Humanities and Social Sciences Communications*, 10(1), 1–11. <https://doi.org/10.1057/s41599-023-01508-1>
- Yildiz, A., & Baltaci, S. (2016). Reflections from the analytic geometry courses based on contextual teaching and learning through. *The Online Journal of New Horizons in Education*, 6(4), 155–166.
- Zeinul, A., & Silmi, J. (2022). Difficulties in learning geometry component in mathematics and active-based learning methods to overcome the difficulties. *Shanlax International Journal of Education*, 10(2), 41–58. <https://doi.org/10.34293/education.v10i2.4299>
- Zengin, Y. (2017). The effects of GeoGebra software on pre-service mathematics teachers' attitudes and views toward proof and proving. *International Journal of Mathematical Education in Science and Technology*, 48(7), 1002–1022. <https://doi.org/10.1080/0020739X.2017.1298855>
- Zulnaidi, H., & Zakaria, E. (2012). The effect of using GeoGebra on conceptual and procedural knowledge of high school mathematics students. *Asian Social Science*, 8(11), 102–106. <https://doi.org/10.5539/ass.v8n11p102>